LESSON MAKER PRO

Square It Up

Rewrite quadratics, then find their solutions.

GRADE 8 · 8A.AF.D.11.a

8A.AF.D.11.a "Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Explain how the quadratic formula is derived from this form."

NAME _____ DATE _____

COMPLETING THE SQUARE

To solve $ax^2 + bx + c = 0$, first divide by a so the coefficient of x^2 is 1. Move the constant to the other side. Then add the square of half the x coefficient to both sides and factor the left side.

WORKED EXAMPLE

Rewrite $x^2 + 18x + 56 = 0$ as a squared equation, then solve.

1. Move the constant: $x^2 + 18x = -56$.
2. Half of 18 is 9, and $9^2 = 81$.
3. Add 81 to both sides: $x^2 + 18x + 81 = 25$, so $(x + 9)^2 = 25$.
4. Take square roots: $x + 9 = \pm 5$, so $x = -4$ or $x = -14$.

Answer: $(x + 9)^2 = 25$; $x = -4$ or $x = -14$

PRACTICE 6 problems

1 Complete the square to rewrite $x^2 + 4x - 5 = 0$ in the form $(x - p)^2 = q$. Then solve.

ANSWER _____

2 Complete the square to rewrite $x^2 - 8x + 3 = 0$ in the form $(x - p)^2 = q$. Then solve.

ANSWER _____

3 Complete the square to rewrite $2x^2 + 12x + 10 = 0$ in the form $(x - p)^2 = q$. Then solve.

ANSWER _____

4 Complete the square to rewrite $3x^2 - 6x + 5 = 0$ in the form $(x - p)^2 = q$. Does it have any real solutions?

ANSWER _____

5 Complete the square to rewrite $x^2 + 3x - 1 = 0$ in the form $(x - p)^2 = q$.

ANSWER _____

6 Complete the square to rewrite $4x^2 - 4x - 3 = 0$ in the form $(x - p)^2 = q$. Then solve.

ANSWER _____

APPLY IT Show your work

1 You are making a rectangular sticker for your laptop. Its width is x centimeters, and its length is x + 5 centimeters. Its area is 36 square centimeters. Complete the square to find the sticker's width and length.

WORK SPACE

ANSWER _____

2 A toy drone starts 5 meters above the ground. Its height, in meters, after x seconds is $-x^2 + 4x + 5$. Complete the square to find when it reaches the ground.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Start with $ax^2 + bx + c = 0$, where a is not 0. Complete the square to get an equation in the form $(x - p)^2 = q$. Use it to derive $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$. Explain when the original equation has no real solutions.

WORK SPACE AND ANSWER

ANSWER KEY # Square It Up

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Complete the square to rewrite $x^2 + 4x - 5 = 0$ in the form $(x - p)^2 = q$. Then solve.	**$(x + 2)^2 = 9$; $x = 1$ or $x = -5$**
2	Complete the square to rewrite $x^2 - 8x + 3 = 0$ in the form $(x - p)^2 = q$. Then solve.	**$(x - 4)^2 = 13$; $x = 4 \pm \sqrt{13}$**
3	Complete the square to rewrite $2x^2 + 12x + 10 = 0$ in the form $(x - p)^2 = q$. Then solve.	**$(x + 3)^2 = 4$; $x = -1$ or $x = -5$**
4	Complete the square to rewrite $3x^2 - 6x + 5 = 0$ in the form $(x - p)^2 = q$. Does it have any real solutions?	**$(x - 1)^2 = -2/3$; no real solutions**
5	Complete the square to rewrite $x^2 + 3x - 1 = 0$ in the form $(x - p)^2 = q$.	**$(x + 3/2)^2 = 13/4$**
6	Complete the square to rewrite $4x^2 - 4x - 3 = 0$ in the form $(x - p)^2 = q$. Then solve.	**$(x - 1/2)^2 = 1$; $x = 3/2$ or $x = -1/2$**

PART 2 · APPLY IT

1. **Width: 4 cm; length: 9 cm** From $x(x + 5) = 36$, get $(x + 5/2)^2 = 169/4$. The solutions are $x = 4$ and $x = -9$; only 4 cm is a possible width.
2. **After 5 seconds** Set the height to 0 and rearrange to get $x^2 - 4x = 5$, so $(x - 2)^2 = 9$. The solutions are $x = 5$ and $x = -1$; only 5 seconds is after the start.

CHALLENGE

Divide by a and move the constant: $x^2 + (b/a)x = -c/a$. Add $b^2/(4a^2)$ to both sides: $(x + b/(2a))^2 = (b^2 - 4ac)/(4a^2)$. Multiply by $4a^2$ to get $(2ax + b)^2 = b^2 - 4ac$. Taking square roots gives $2ax + b = \pm\sqrt{b^2 - 4ac}$, so $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$. There are no real solutions when $b^2 - 4ac < 0$.

Accept equivalent squared forms and solution orders. In context problems, students should reject negative lengths and times before the stated start.