LESSON MAKER PRO

Patterns With Power

Write rules for patterns that add or multiply.

GRADE 8 · 8A.AF.H.24.a

8A.AF.H.24.a "Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions."

NAME _____ DATE _____

TWO KINDS OF GROWTH

An arithmetic sequence adds the same amount, d, each step. Its explicit rule is $a_n = a_1 + (n - 1)d$, its recursive rule adds d to the previous term, and its values follow a linear function. A geometric sequence multiplies by the same number, r, each step. Its explicit rule is $a_n = a_1 r^{n-1}$, its recursive rule multiplies the previous term by r, and its values follow an exponential function.

WORKED EXAMPLE

For 13, 19, 25, 31, write explicit and recursive rules. Then write the matching function rule.

1. The common difference is 19 - 13 = 6.
2. Explicit: $a_n = 13 + 6(n - 1) = 6n + 7$.
3. Recursive: $a_1 = 13$; $a_n = a_{n-1} + 6$ for $n \geq 2$.
4. Replace n with x: y = 6x + 7. This function is linear.

Answer: $a_n = 6n + 7$; $a_1 = 13$ and $a_n = a_{n-1} + 6$ for $n \geq 2$; linear function $y = 6x + 7$.

PRACTICE 6 problems

1 For 2, 5, 8, 11, write an explicit rule and identify the function type.

ANSWER _____

2 For 40, 32, 24, 16, write a recursive rule and find the fifth term.

ANSWER _____

3 For 5, 10, 20, 40, write an explicit rule and identify the function type.

ANSWER _____

4 For 96, 48, 24, 12, write a recursive rule and find the sixth term.

ANSWER _____

5 An arithmetic sequence has the rule $a_n = 30 - 4(n - 1)$. Find the fourth term and write the matching linear function as y = mx + b.

ANSWER _____

6 A geometric sequence has the rule $a_n = 2 \cdot 4^{n-1}$. Write a recursive rule and find the fourth term.

ANSWER _____

APPLY IT Show your work

1 On day 1 of a school sticker swap, Maya has 42 stickers. She gets 15 more stickers each day. Write explicit and recursive rules for the number she has on day n. How many does she have on day 8, and is the matching function linear or exponential?

WORK SPACE

ANSWER _____

2 A video clip is shared 3 times in its first hour. The number of shares in each later hour is twice the number in the previous hour. Write explicit and recursive rules for the shares during hour n. How many shares occur during hour 6, and is the matching function linear or exponential?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Two sequences both begin 2, 4. One is arithmetic, and the other is geometric. Write an explicit rule for each. What is the first term number n at which the geometric term exceeds the arithmetic term by at least 10? Show how you know it is the first.

WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For 2, 5, 8, 11, write an explicit rule and identify the function type.	**$a_n = 2 + 3(n - 1) = 3n - 1$; linear.**
2	For 40, 32, 24, 16, write a recursive rule and find the fifth term.	**$a_1 = 40$; $a_n = a_{n-1} - 8$ for $n \geq 2$. The fifth term is 8.**
3	For 5, 10, 20, 40, write an explicit rule and identify the function type.	**$a_n = 5 \cdot 2^{n-1}$; exponential.**
4	For 96, 48, 24, 12, write a recursive rule and find the sixth term.	**$a_1 = 96$; $a_n = a_{n-1}/2$ for $n \geq 2$. The sixth term is 3.**
5	An arithmetic sequence has the rule $a_n = 30 - 4(n - 1)$. Find the fourth term and write the matching linear function as $y = mx + b$.	**The fourth term is 18; $y = -4x + 34$.**
6	A geometric sequence has the rule $a_n = 2 \cdot 4^{n-1}$. Write a recursive rule and find the fourth term.	**$a_1 = 2$; $a_n = 4a_{n-1}$ for $n \geq 2$. The fourth term is 128.**

PART 2 · APPLY IT

1. **$a_n = 42 + 15(n - 1)$; $a_1 = 42$ and $a_n = a_{n-1} + 15$ for $n \geq 2$; 147 stickers on day 8; linear.** Add 15 for each of the seven days after day 1: $42 + 15(7) = 147$. A constant daily increase produces a linear function.
2. **$a_n = 3 \cdot 2^{n-1}$; $a_1 = 3$ and $a_n = 2a_{n-1}$ for $n \geq 2$; 96 shares during hour 6; exponential.** Hour 6 is five doublings after hour 1, so $3 \cdot 2^5 = 96$. A constant multiplier produces an exponential function.

CHALLENGE

Arithmetic: $a_n = 2n$. Geometric: $a_n = 2^n$. The first is $n = 5$: at $n = 4$, the difference is $16 - 8 = 8$; at $n = 5$, it is $32 - 10 = 22$. The differences at $n = 1$, 2, and 3 are 0, 0, and 2, so none reaches 10.

Accept equivalent simplified or unsimplified explicit rules. Recursive rules must give the first term and apply to $n \geq 2$.