LESSON MAKER PRO

Square the Equation

Transform quadratic equations, then find their solutions.

HIGH SCHOOL · A1P.AF.C.9.a

A1P.AF.C.9.a "Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Explain how the quadratic formula is derived from this form."

NAME _____ DATE _____

COMPLETE THE SQUARE

For an equation beginning with ax^2, first divide every term by a. Move the constant to the other side, then add the square of half the x coefficient to both sides. Factor the perfect square before taking square roots.

WORKED EXAMPLE

Complete the square and solve $3x^2 + 12x - 15 = 0$.

1. Divide by 3: $x^2 + 4x - 5 = 0$.
2. Move the constant: $x^2 + 4x = 5$.
3. Add $(4/2)^2 = 4$ to both sides: $x^2 + 4x + 4 = 9$.
4. Factor and take square roots: $(x + 2)^2 = 9$, so $x + 2 = \pm 3$.

Answer: $(x + 2)^2 = 9$; $x = 1$ or $x = -5$.

PRACTICE 6 problems

1 Complete the square. Write an equivalent squared equation, then solve for real x: $x^2 - 10x + 21 = 0$.

ANSWER _____

2 Complete the square. Write an equivalent squared equation, then solve for real x: $x^2 + 8x + 5 = 0$.

ANSWER _____

3 Complete the square. Write an equivalent squared equation, then solve for real x: $2x^2 - 8x + 1 = 0$.

ANSWER _____

4 Complete the square. Write an equivalent squared equation, then solve for real x: $-x^2 + 4x + 1 = 0$.

ANSWER _____

5 Complete the square. Write an equivalent squared equation, then find any real solutions: $4x^2 + 4x + 5 = 0$.

ANSWER _____

6 Complete the square. Write an equivalent squared equation, then solve for real x: $5x^2 - 10x - 1 = 0$.

ANSWER _____

APPLY IT Show your work

1 During a video shoot, a ball is tossed upward from a 5-meter balcony. Its height in meters after x seconds is $h = -5x^2 + 20x + 5$. When does it hit the ground? Complete the square and give the exact time.

WORK SPACE

ANSWER _____

2 A student concert sells tickets at x dollars each. A model predicts that 40 - 2x tickets will sell, making the revenue $x(40 - 2x)$ dollars. What ticket prices give $192 in revenue? Complete the square and give both prices.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Start with $ax^2 + bx + c = 0$, where a is not 0 and $b^2 - 4ac$ is at least 0. Complete the square to write an equivalent equation in squared form. Then explain how taking square roots gives the quadratic formula.

WORK SPACE AND ANSWER

ANSWER KEY

Square the Equation

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Complete the square. Write an equivalent squared equation, then solve for real x: $x^2 - 10x + 21 = 0$.	**$(x - 5)^2 = 4$; x = 3 or x = 7.**
2	Complete the square. Write an equivalent squared equation, then solve for real x: $x^2 + 8x + 5 = 0$.	**$(x + 4)^2 = 11$; x = -4 ± sqrt(11).**
3	Complete the square. Write an equivalent squared equation, then solve for real x: $2x^2 - 8x + 1 = 0$.	**$(x - 2)^2 = 7/2$; x = 2 ± sqrt(14)/2.**
4	Complete the square. Write an equivalent squared equation, then solve for real x: $-x^2 + 4x + 1 = 0$.	**$(x - 2)^2 = 5$; x = 2 ± sqrt(5).**
5	Complete the square. Write an equivalent squared equation, then find any real solutions: $4x^2 + 4x + 5 = 0$.	**$(x + 1/2)^2 = -1$; no real solutions.**
6	Complete the square. Write an equivalent squared equation, then solve for real x: $5x^2 - 10x - 1 = 0$.	**$(x - 1)^2 = 6/5$; x = 1 ± sqrt(30)/5.**

PART 2 · APPLY IT

1. **$(x - 2)^2 = 5$; the ball hits the ground after 2 + sqrt(5) seconds.** Set h = 0, divide by -5, and complete the square. The other solution, 2 - sqrt(5), is negative, so it is not a time after the toss.
2. **$(x - 10)^2 = 4$; the ticket prices are $8 and $12.** Set $x(40 - 2x) = 192$, rearrange to $x^2 - 20x + 96 = 0$, and complete the square. Both prices give $192 in revenue.

CHALLENGE

Divide by a and move the constant: $x^2 + (b/a)x = -c/a$. Add $b^2/(4a^2)$ to both sides and factor: $(x + b/(2a))^2 = (b^2 - 4ac)/(4a^2)$. Taking both square roots and isolating x gives $x = (-b ± sqrt(b^2 - 4ac))/(2a)$.

Accept equivalent exact radical forms and either order for two solutions. In the ball problem, only the positive time answers the question.