LESSON MAKER PRO

When Growth Wins

Compare patterns in tables and equations.

HIGH SCHOOL · A1P.AF.H.26

A1P.AF.H.26 "Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically."

NAME _____ DATE _____

THE BIG IDEA

A linear pattern adds a fixed amount, while an exponential pattern multiplies by a fixed factor. A quadratic pattern has constant second differences. Extend a table to find when an exponential value passes another value, then check later rows to see whether its lead grows.

WORKED EXAMPLE

Use the table. When does E first exceed L? Does E's lead grow in the next row?

$E = 3 \cdot 3^N$, $L = 8N + 5$

n	E	L
0	3	5
1	9	13
2	27	21
3	81	29

1. At n = 0 and n = 1, E is less than L.
2. At n = 2, E = 27 and L = 21, so E first passes L.
3. E's lead grows from 6 at n = 2 to 52 at n = 3.

Answer: E first exceeds L at n = 2. Its lead grows in the next row.

PRACTICE 4 problems

1 For $E = 2^n$ and $L = n + 6$, find the first n from 0 through 4 when $E > L$.

ANSWER _____

2 Check n = 0, 1, and 2. When does $E = 4^n$ first exceed $L = 6n + 2$?

ANSWER _____

3 Compare $E = 3^n$ and $Q = n^2 + 9$. Which is larger at n = 2? At n = 3?

ANSWER _____

4 Let $E = 2 \cdot 2^n$ and $Q = n^2 + 7$. Are they equal at n = 3? Which is larger at n = 4?

ANSWER _____

APPLY IT Show your work

1 Two clips gain views as shown. At which hour shown does Clip A first pass Clip B? Which clip doubles each hour?

CLIP VIEWS

Hour	Clip A	Clip B
1	4	12
2	8	15
3	16	18
4	32	21

ANSWER _____

2 A game offers two round bonuses. At which round shown does the doubling bonus first pass the square bonus? How do the square bonus's differences support its name?

BONUS POINTS

Round	Doubling	Square
2	4	6
3	8	11
4	16	18
5	32	27

ANSWER _____

CHALLENGE One step up

For $E = 2^n$ and $Q = n^2$, both equal 16 at n = 4. Find both values at n = 5 and n = 6. What do these rows show about judging the patterns from n = 4 alone?

WORK SPACE AND ANSWER

ANSWER KEY

When Growth Wins

HIGH SCHOOL · A1P.AF.H.26

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $E = 2^n$ and $L = n + 6$, find the first n from 0 through 4 when $E > L$.	**n = 4**
2	Check n = 0, 1, and 2. When does $E = 4^n$ first exceed $L = 6n + 2$?	**n = 2**
3	Compare $E = 3^n$ and $Q = n^2 + 9$. Which is larger at n = 2? At n = 3?	**At n = 2, Q is larger: 13 > 9. At n = 3, E is larger: 27 > 18.**
4	Let $E = 2 \cdot 2^n$ and $Q = n^2 + 7$. Are they equal at n = 3? Which is larger at n = 4?	**At n = 3, both equal 16. At n = 4, E is larger: 32 > 23.**

PART 2 · APPLY IT

1. **Hour 4; Clip A doubles each hour.** At hour 3, A has 16 views and B has 18. At hour 4, A has 32 and B has 21; A's values double from row to row.
2. **Round 5. The square bonus increases by 5, 7, then 9 points, giving constant second differences of 2.** The doubling bonus is below the square bonus through round 4, then passes it at round 5. The square bonus's first differences rise by 2 each time.

CHALLENGE

At n = 5, E = 32 and Q = 25. At n = 6, E = 64 and Q = 36. The tie at n = 4 does not continue: E passes Q and has a larger lead at n = 6.

Accept comparisons stated in words or with inequality symbols. For the challenge, students should identify the tie, then use both later rows to show E pulling ahead.