LESSON MAKER PRO

Mirror Graphs

Track how key features change in inverse functions.

HIGH SCHOOL · A2S.AF.H.20.f

A2S.AF.H.20.f "Compare the graphs of inverse functions and the relationships between their key features, including but not limited to quadratic, square root, exponential, and logarithmic functions."

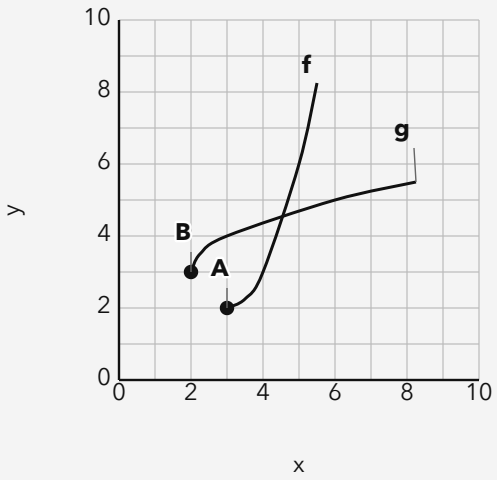
NAME _____ DATE _____

SWAP AND REFLECT

Inverse functions exchange inputs and outputs, so a point (a, b) on one graph becomes (b, a) on the other. Their graphs reflect across the line y = x. The domain and range also switch, and a quadratic needs a restricted domain to have an inverse function.

WORKED EXAMPLE

Use the graph to find the endpoints of f and g. Then state the domain and range of g.

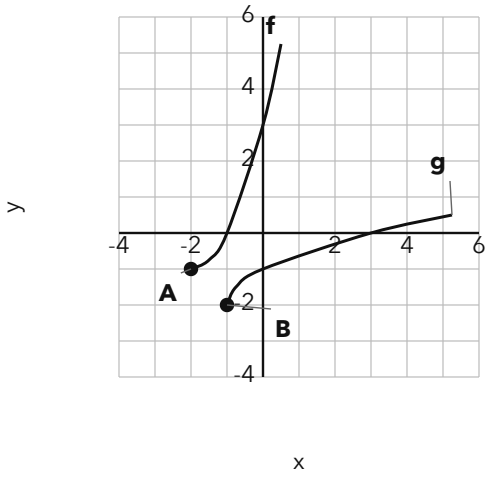


1. Read A as the endpoint (3, 2) of f.
2. Swap its coordinates to get B, the endpoint (2, 3) of g.
3. Read the x-values of g for its domain and its y-values for its range.

Answer: The endpoints are (3, 2) for f and (2, 3) for g. The domain of g is x >= 2, and its range is y >= 3.

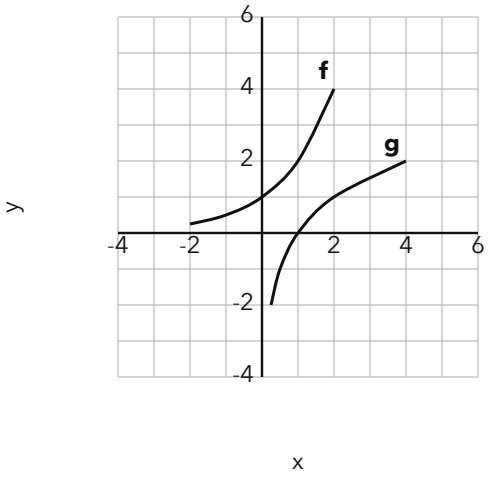
PRACTICE 6 problems

1 The graphed functions f and g are inverses. What are their endpoints?



ANSWER _____

2 The graphed functions f and g are inverses. Find the y-intercept of f and the x-intercept of g. How are they related?



ANSWER _____

3 If $f(x) = 4^x$, what is $f^{-1}(16)$?

ANSWER _____

4 Let $f(x) = (x - 5)^2 + 4$ for $x >= 5$. Give the domain and the endpoint of f^{-1}.

ANSWER _____

5 Let $f(x) = \text{sqrt}(x + 6) - 3$. Find the equation and domain of f^{-1}.

ANSWER _____

6 Let $f(x) = \log_3(x) + 2$. The point (9, 4) is on f. Give the matching point on f^{-1}, and name the horizontal asymptote of f^{-1}.

ANSWER _____

APPLY IT Show your work

1 A playlist starts with 6 listeners and doubles its audience each day. The model is $L(t) = 6(2^t)$ for $t >= 0$, where t is days since the playlist launched. What does $L^{-1}(48)$ mean, and what is its value? What is the smallest input allowed for L^{-1}?

WORK SPACE

ANSWER _____

2 During the rise of a game character's jump, its height is $h(t) = 16 - (t - 4)^2$ feet for $0 <= t <= 4$ seconds. At what time does the character reach 7 feet? What does the endpoint (16, 4) of the inverse graph mean?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student says the inverse of $f(x) = (x + 1)^2 - 4$ for $x <= -1$ is $g(x) = \sqrt{x + 4} - 1$. Explain why this is wrong. Give the correct inverse, its domain, and its endpoint.

WORK SPACE AND ANSWER

ANSWER KEY

Mirror Graphs

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The graphed functions f and g are inverses. What are their endpoints?	**The endpoint of f is (-2, -1). The endpoint of g is (-1, -2).**
2	The graphed functions f and g are inverses. Find the y-intercept of f and the x-intercept of g. How are they related?	**The y-intercept of f is (0, 1), and the x-intercept of g is (1, 0). Their coordinates are switched.**
3	If $f(x) = 4^x$, what is $f^{-1}(16)$?	**2**
4	Let $f(x) = (x - 5)^2 + 4$ for $x >= 5$. Give the domain and the endpoint of f^{-1}.	**The domain is x >= 4. The endpoint is (4, 5).**
5	Let $f(x) = \sqrt{x + 6} - 3$. Find the equation and domain of f^{-1}.	**$f^{-1}(x) = (x + 3)^2 - 6$, with domain x >= -3.**
6	Let $f(x) = \log_3(x) + 2$. The point (9, 4) is on f. Give the matching point on f^{-1}, and name the horizontal asymptote of f^{-1}.	**The matching point is (4, 9). The horizontal asymptote is y = 0.**

PART 2 · APPLY IT

1. **$L^{-1}(48)$ is the number of days until the model reaches 48 listeners: 3 days. The smallest input to L^{-1} is 6 listeners.** Solve $6(2^t) = 48$ to get $2^t = 8$ and t = 3. Since t >= 0, the model starts at 6 listeners.
2. **The character reaches 7 feet at 1 second. The inverse endpoint (16, 4) means the character reaches its highest height, 16 feet, at 4 seconds.** Set $7 = 16 - (t - 4)^2$. This gives $t - 4 = -3$ on the stated time interval, so t = 1.

CHALLENGE

The restriction x <= -1 means x + 1 <= 0, so solving for the inverse requires the negative square root. The correct inverse is $f^{-1}(x) = -1 - \sqrt{x + 4}$, with domain x >= -4 and endpoint (-4, -1).

Accept interval notation for domains and ranges. For the challenge, accept any explanation that uses the restricted branch of the quadratic to justify the negative square root.