LESSON MAKER PRO

Means in Motion

See how sample size changes the spread of sample means.

HIGH SCHOOL · A2S.DSP.H.31

A2S.DSP.H.31 "Evaluate the effect of sample size on the expected variability in the sampling distribution of a sample statistic. a. Simulate a sampling distribution of sample means from a population with a known distribution..."

NAME _____ DATE _____

THE SPREAD RULE

For samples of size n from a population with standard deviation sigma, the standard deviation of the sample means is sigma/sqrt(n). Larger samples produce less spread among their means. A random simulation usually comes close to this value, while listing every possible sample with replacement gives the exact value.

WORKED EXAMPLE

A population has a standard deviation of 35. Find the standard deviation of the means of samples of size 25.

1. Use SD(sample means) = sigma/sqrt(n).
2. Substitute: 35/sqrt(25).
3. Since sqrt(25) = 5, divide: 35/5 = 7.

Answer: 7

PRACTICE 6 problems

1 A population has a standard deviation of 12. Find the standard deviation of the means of samples of size 16.

ANSWER _____

2 A population has a standard deviation of 18. Find the standard deviation of the means of samples of size 36.

ANSWER _____

3 A population has a standard deviation of 24. Find the standard deviation of the sample means when n = 16 and when n = 64.

ANSWER _____

4 A population has a standard deviation of 20. What sample size n makes the standard deviation of the sample means 5?

ANSWER _____

5 A population has a standard deviation of 15. Find the standard deviation of the sample means when n = 1 and when n = 9.

ANSWER _____

6 Simulate every ordered sample of size 2 with replacement from the equally likely population {0, 8}. List all four sample means, then find their standard deviation.

ANSWER _____

APPLY IT Show your work

1 Upload times for videos on a school media site have a population standard deviation of 48 seconds. Find the standard deviation of the mean upload time for samples of 16 videos and for samples of 64 videos. Which sample size gives less spread?

WORK SPACE

ANSWER _____

2 The lengths of songs in a music library have a population standard deviation of 30 seconds. For samples of 36 songs, what is the standard deviation of the mean song length? What is the smallest sample size that makes this standard deviation at most 3 seconds?

WORK SPACE

ANSWER _____

CHALLENGE One step up

All 16 equally likely ordered samples of size 4 are drawn with replacement from the population {-8, 8}. The table shows their sample means. Use the average of the squared deviations to find the standard deviation of these 16 means. Then check it with sigma/sqrt(n). Why do the results agree?

ALL SAMPLE MEANS

Sample mean	Frequency
-8	1
-4	4
0	6
4	4
8	1

WORK SPACE AND ANSWER

Means in Motion

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A population has a standard deviation of 12. Find the standard deviation of the means of samples of size 16.	**3**
2	A population has a standard deviation of 18. Find the standard deviation of the means of samples of size 36.	**3**
3	A population has a standard deviation of 24. Find the standard deviation of the sample means when n = 16 and when n = 64.	**6 when n = 16; 3 when n = 64.**
4	A population has a standard deviation of 20. What sample size n makes the standard deviation of the sample means 5?	**16**
5	A population has a standard deviation of 15. Find the standard deviation of the sample means when n = 1 and when n = 9.	**15 when n = 1; 5 when n = 9.**
6	Simulate every ordered sample of size 2 with replacement from the equally likely population {0, 8}. List all four sample means, then find their standard deviation.	**Sample means: 0, 4, 4, 8. Standard deviation: sqrt(8), or 2sqrt(2).**

PART 2 · APPLY IT

1. **12 seconds for 16 videos; 6 seconds for 64 videos. Samples of 64 give less spread.** Compute 48/sqrt(16) = 12 and 48/sqrt(64) = 6. Compare the two results.
2. **5 seconds for samples of 36; at least 100 songs for a standard deviation of at most 3 seconds.** Compute 30/sqrt(36) = 5. Solve 30/sqrt(n) ≤ 3, so sqrt(n) ≥ 10 and n ≥ 100.

CHALLENGE

The mean is 0, so the average squared deviation is $[2(8^2) + 8(4^2)]/16 = 16$, giving a standard deviation of 4. The population standard deviation is 8, and 8/sqrt(4) = 4; the results agree because the table includes every equally likely ordered sample.

Accept sqrt(8) or 2sqrt(2) for practice problem 6. Students should distinguish the exact spread from listing every possible sample from the approximate spread of a random simulation.