LESSON MAKER PRO

Where Means Gather

Find the center and shape of sampling distributions.

HIGH SCHOOL · A2S.DSP.I.32.a

A2S.DSP.I.32.a "Verify that a sampling distribution is centered at the population mean and approximately normal if the sample size is large enough."

NAME _____ DATE _____

THE BIG IDEA

For independent random samples, the mean of all sample means equals the population mean. If the population is normal, the sample means have a normal distribution at any sample size. Otherwise, a sample size of at least 30 is a common rule for an approximately normal distribution of sample means.

WORKED EXAMPLE

A skewed population has a mean of 17. Independent random samples of size 49 are taken. Find the center of the sample means and describe their approximate shape.

1. The center of the sample means equals the population mean, 17.
2. The sample size, 49, is at least 30.
3. The distribution of sample means is approximately normal.

Answer: Center: 17. Shape: approximately normal.

PRACTICE 6 problems

1 A skewed population has a mean of 8. Independent random samples of size 36 are taken. What is the center of the sample means? Is their distribution approximately normal?

ANSWER _____

2 A normal population has a mean of 12. Independent random samples of size 9 are taken. What is the center of the sample means? Is their distribution normal?

ANSWER _____

3 A strongly skewed population has a mean of 5. Independent random samples of size 16 are taken. What is the center of the sample means? Does the sample-size rule support a normal approximation?

ANSWER _____

4 The table lists every possible sample mean from independent samples of size 2. The population mean is 4. Find the mean of the sample means. Does it match the population mean?

SAMPLING DISTRIBUTION

Sample mean	Frequency
2	1
4	2
6	1

ANSWER _____

5 A strongly skewed population has a mean of 11. Compare independent random samples of size 10 and size 40. Give the center for each sampling distribution. Which one does the sample-size rule say is approximately normal?

ANSWER _____

6 A population has the values and frequencies shown. Independent random samples of size 64 are drawn with replacement. Find the center of the sample means. Is their distribution approximately normal?

POPULATION VALUES

Value	Frequency
1	1
3	2
5	1

ANSWER _____

APPLY IT Show your work

1 Students' daily music-streaming times are skewed, with a population mean of 80 minutes. A school takes independent random samples of 45 students. One sample has a mean of 86 minutes. Find the center and approximate shape of the distribution of all such sample means. Does the observed mean of 86 minutes change the center?

WORK SPACE

ANSWER _____

2 In a game, a player wins either $3 or $11, with each amount equally likely on each independent play. List the possible means and their probabilities for samples of two plays. Find the center of this sampling distribution. Is it normal?

WORK SPACE

ANSWER _____

CHALLENGE One step up

The table shows the points awarded on one play of a game. Each row's frequency shows how many of four equally likely outcomes award that number of points. Plays are independent. For samples of 2 plays and samples of 50 plays, find the center of each distribution of sample means. Which distribution is approximately normal? Explain why the other is not normal.

POINTS PER PLAY

Points	Frequency
0	1
4	2
8	1

WORK SPACE AND ANSWER

Where Means Gather

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A skewed population has a mean of 8. Independent random samples of size 36 are taken. What is the center of the sample means? Is their distribution approximately normal?	**Center: 8. Yes, it is approximately normal.**
2	A normal population has a mean of 12. Independent random samples of size 9 are taken. What is the center of the sample means? Is their distribution normal?	**Center: 12. Yes, it is normal.**
3	A strongly skewed population has a mean of 5. Independent random samples of size 16 are taken. What is the center of the sample means? Does the sample-size rule support a normal approximation?	**Center: 5. No, 16 is below 30, so the rule does not support a normal approximation.**
4	The table lists every possible sample mean from independent samples of size 2. The population mean is 4. Find the mean of the sample means. Does it match the population mean?	**The mean is (2 + 4 + 4 + 6)/4 = 4. Yes, it matches the population mean.**
5	A strongly skewed population has a mean of 11. Compare independent random samples of size 10 and size 40. Give the center for each sampling distribution. Which one does the sample-size rule say is approximately normal?	**Both have center 11. The distribution for samples of size 40 is approximately normal by the sample-size rule.**
6	A population has the values and frequencies shown. Independent random samples of size 64 are drawn with replacement. Find the center of the sample means. Is their distribution approximately normal?	**Center: (1 + 3 + 3 + 5)/4 = 3. Yes, the distribution is approximately normal.**

PART 2 · APPLY IT

1. **The center is 80 minutes, and the distribution is approximately normal. The observed mean of 86 minutes does not change the center.** The center equals the population mean of 80 minutes. Since 45 is at least 30, the sample means are approximately normal despite the skewed population.
2. **The possible means are $3 with probability 1/4, $7 with probability 1/2, and $11 with probability 1/4. The center is $7, and the distribution is not normal.** The four equally likely pairs are (3, 3), (3, 11), (11, 3), and (11, 11). Their means average to $7; the sampling distribution has only three possible values, so it is not normal.

CHALLENGE

The population mean is (0 + 4 + 4 + 8)/4 = 4, so both sampling distributions have center 4. The distribution for samples of 50 is approximately normal by the sample-size rule. For samples of 2, the mean can only be 0, 2, 4, 6, or 8, so its distribution is not normal.

Accept equivalent calculations of a weighted mean. A sample size below 30 does not rule out an approximately normal shape in general, but the three-value distribution in the game problem and the five-value distribution in the challenge are not normal.