LESSON MAKER PRO

Means in Motion

Build a sampling distribution and use it to estimate a mean.

HIGH SCHOOL · A2S.DSP.I.32

A2S.DSP.I.32 "Produce a sampling distribution by repeatedly selecting samples of the same size from a given population or from a population simulated by bootstrapping (resampling with replacement from an observed sample). Do initial examples by hand..."

NAME _____ DATE _____

FROM SAMPLES TO ESTIMATES

Draw many samples of the same size and record each sample mean. For samples from a known population, the sampling distribution is centered at the population mean and is often roughly normal when the samples are large enough. About 95% of sample means fall within two sampling-distribution standard deviations of the population mean, so an approximate 95% confidence interval is the observed mean plus or minus two of those standard deviations.

WORKED EXAMPLE

The population is 11 and 19. Draw two values with replacement. The table shows every equally likely sample. Find the center of the sample means and compare it with the population mean.

ALL POSSIBLE SAMPLES

First draw	Second draw	Sample mean
11	11	11
11	19	15
19	11	15
19	19	19

1. Average the sample means: (11 + 15 + 15 + 19)/4 = 15.
2. Average the population values: (11 + 19)/2 = 15.

Answer: Both centers are 15.

PRACTICE 4 problems

1 The population is 0 and 8. Draw two values with replacement. List all four equally likely sample means, then find their mean.

ANSWER _____

2 A sampling distribution has mean 50 and standard deviation 5. Give the interval containing about 95% of its sample means.

ANSWER _____

3 An observed mean is 72. The standard deviation of the sampling distribution is 4. Find an approximate 95% confidence interval.

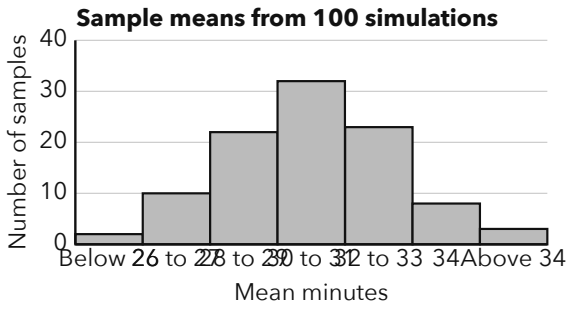
ANSWER _____

4 Of 200 sample means, 188 are within two sampling-distribution standard deviations of the population mean. What percent is this? Is it close to 95%?

ANSWER _____

APPLY IT Show your work

1 A school simulates 100 samples of 40 students' daily music minutes. The population mean is 30 minutes, and the sampling-distribution standard deviation is 2 minutes. Use the histogram. What percent of means are within two standard deviations? Is the shape roughly bell-shaped?



ANSWER _____

2 An after-school club surveys students' homework time. The observed mean is 48 minutes. A simulation repeatedly resamples the survey responses with replacement, using the original sample size each time. The standard deviation of the simulated means is 3 minutes. Give an approximate 95% confidence interval and interpret it.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Two simulations draw samples from the same population, whose mean is 60. Samples of size 25 produce a sampling-distribution standard deviation of 6. Samples of size 100 produce one of 3. Give the range containing about 95% of sample means for each size. Which range is narrower, and why?

WORK SPACE AND ANSWER

ANSWER KEY

Means in Motion

HIGH SCHOOL · A2S.DSP.I.32

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The population is 0 and 8. Draw two values with replacement. List all four equally likely sample means, then find their mean.	**Sample means: 0, 4, 4, 8. Their mean is 4.**
2	A sampling distribution has mean 50 and standard deviation 5. Give the interval containing about 95% of its sample means.	**40 to 60**
3	An observed mean is 72. The standard deviation of the sampling distribution is 4. Find an approximate 95% confidence interval.	**64 to 80**
4	Of 200 sample means, 188 are within two sampling-distribution standard deviations of the population mean. What percent is this? Is it close to 95%?	**94%; yes, it is close to 95%.**

PART 2 · APPLY IT

1. **95%; yes, the shape is roughly bell-shaped.** Two standard deviations give 30 ± 4, or 26 to 34 minutes. The five bins in that range contain 10 + 22 + 32 + 23 + 8 = 95 of the 100 means.
2. **42 to 54 minutes; we are about 95% confident that the mean homework time for the students the club surveyed from is in this interval.** Calculate 48 ± 2(3) = 48 ± 6. The interval estimates a population mean, not the range of individual homework times.

CHALLENGE

Size 25: 48 to 72. Size 100: 54 to 66. The size 100 range is narrower because its sample means vary less.

Accept intervals written with brackets or inequalities. The histogram is only approximately bell-shaped; students need not claim it is perfectly normal.