LESSON MAKER PRO

The Missing Ratio

Prove the identity, then use it to find exact values.

HIGH SCHOOL · A2S.GM.B.36

A2S.GM.B.36 "Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios."

NAME _____ DATE _____

WHY THE IDENTITY WORKS

For an acute angle θ in a right triangle, sin θ and cos θ each have the hypotenuse c as their denominator. If the legs are a and b, then $\sin^2\theta + \cos^2\theta = (a^2 + b^2)/c^2 = 1$ by the Pythagorean theorem. The identity also holds in every quadrant, so use the quadrant to choose the sign of a missing ratio.

WORKED EXAMPLE

If $\sin\theta = 8/17$ and θ is in quadrant I, find $\cos\theta$.

1. $\cos^2\theta = 1 - \sin^2\theta$
2. $\cos^2\theta = 1 - 64/289 = 225/289$
3. Cosine is positive in quadrant I, so $\cos\theta = 15/17$.

Answer: 15/17

PRACTICE 6 problems

1 If $\sin\theta = 3/5$ and θ is in quadrant I, find $\cos\theta$.

ANSWER _____

2 If $\cos\theta = 5/13$ and θ is in quadrant IV, find $\sin\theta$.

ANSWER _____

3 If $\sin\theta = -7/25$ and θ is in quadrant III, find $\cos\theta$.

ANSWER _____

4 If $\cos\theta = -9/41$ and θ is in quadrant II, find $\sin\theta$.

ANSWER _____

5 If $\sin\theta = 1/2$ and θ is in quadrant II, find $\cos\theta$.

ANSWER _____

6 If $\sin^2\theta = 16/25$ and $\cos\theta$ is negative, find $\cos^2\theta$ and $\cos\theta$.

ANSWER _____

APPLY IT Show your work

1 A drone travels 29 m along a straight flight path and gains 20 m in altitude. Let θ be the angle of its path above level ground. Find cos θ and the drone's horizontal travel.

WORK SPACE

ANSWER _____

2 In a game, a character moves 37 units along a straight path in a direction θ in quadrant II. Its horizontal change is -12 units. Find sin θ and its vertical change.

WORK SPACE

ANSWER _____

CHALLENGE One step up

If sin θ + cos θ = 1/3, find sin θ cos θ. Show how the Pythagorean identity proves your answer.

WORK SPACE AND ANSWER

ANSWER KEY # The Missing Ratio

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	If $\sin\theta = 3/5$ and θ is in quadrant I, find $\cos\theta$.	**4/5**
2	If $\cos\theta = 5/13$ and θ is in quadrant IV, find $\sin\theta$.	**-12/13**
3	If $\sin\theta = -7/25$ and θ is in quadrant III, find $\cos\theta$.	**-24/25**
4	If $\cos\theta = -9/41$ and θ is in quadrant II, find $\sin\theta$.	**40/41**
5	If $\sin\theta = 1/2$ and θ is in quadrant II, find $\cos\theta$.	**-sqrt(3)/2**
6	If $\sin^2\theta = 16/25$ and $\cos\theta$ is negative, find $\cos^2\theta$ and $\cos\theta$.	**$\cos^2\theta = 9/25$; $\cos\theta = -3/5$**

PART 2 · APPLY IT

1. **cos θ = 21/29; horizontal travel = 21 m** The horizontal travel is sqrt($29^2 - 20^2$) = 21 m. Divide 21 by the path length, 29, to find cos θ.
2. **sin θ = 35/37; vertical change = 35 units** The vertical change is positive in quadrant II and equals sqrt($37^2 - 12^2$) = 35 units. Divide 35 by 37 to find sin θ.

CHALLENGE

$\sin\theta\cos\theta = -4/9$. Squaring the given equation gives $1/9 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1 + 2\sin\theta\cos\theta$. Thus $2\sin\theta\cos\theta = -8/9$, so $\sin\theta\cos\theta = -4/9$.

Accept equivalent exact forms, such as -sqrt(3)/2 written as -1/2 times sqrt(3). In problem 6, both the squared ratio and the signed ratio are required.