LESSON MAKER PRO

Matrix Undo

Find the matrices that cancel or reverse a square matrix.

HIGH SCHOOL · A2S.NQ.B.5.a

A2S.NQ.B.5.a "Find the additive and multiplicative inverses of square matrices, using technology as appropriate."

NAME _____ DATE _____

TWO KINDS OF INVERSE

The additive inverse of a matrix changes the sign of every entry, so the two matrices add to a zero matrix. For a 2 by 2 matrix [[a, b], [c, d]], the multiplicative inverse is [[d, -b], [-c, a]] divided by ad - bc. If ad - bc is 0, there is no multiplicative inverse.

WORKED EXAMPLE

Find both inverses of A = [[7, 2], [5, 3]].

1. Change every entry's sign: -A = [[-7, -2], [-5, -3]].
2. Find the determinant: 7(3) - 2(5) = 11.
3. Swap the diagonal entries and change the other signs: [[3, -2], [-5, 7]].
4. Divide every entry by 11 to find the multiplicative inverse.

Answer: Additive inverse: [[-7, -2], [-5, -3]]. Multiplicative inverse: [[3/11, -2/11], [-5/11, 7/11]].

PRACTICE 6 problems

1 Find the additive inverse of [[4, -1], [0, 6]].

ANSWER _____

2 Find the additive inverse of [[-8, 0], [9, -4]].

ANSWER _____

3 Find the multiplicative inverse of [[1, 4], [0, 1]].

ANSWER _____

4 Find the multiplicative inverse of [[4, 1], [1, 0]].

ANSWER _____

5 Does [[1, 4], [-1, -4]] have a multiplicative inverse? Use its determinant to explain.

ANSWER _____

6 Find both inverses of [[-1, 0], [0, 4]].

ANSWER _____

APPLY IT Show your work

1 A game studio uses a color filter that sends original color values [r, b] to new values [R, B] using the matrix [[1, 1], [0, 1]]. One pixel has new values [14, 6]. Find the filter matrix's multiplicative inverse, then use it to find the original values [r, b].

WORK SPACE

ANSWER _____

2 At a school concert, student tickets cost $4 and adult tickets cost $6. The matrix [[1, 1], [4, 6]] sends [student tickets, adult tickets] to [total tickets, total dollars]. The booth sold 10 tickets for $48. Find the matrix's multiplicative inverse, then use it to find how many of each ticket sold.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For A = [[4, x], [1, 4]], find the value of x that makes the multiplicative inverse impossible. At that value of x, find the additive inverse of A. Explain why one inverse exists but the other does not.

WORK SPACE AND ANSWER

ANSWER KEY

Matrix Undo

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the additive inverse of [[4, -1], [0, 6]].	**[[-4, 1], [0, -6]]**
2	Find the additive inverse of [[-8, 0], [9, -4]].	**[[8, 0], [-9, 4]]**
3	Find the multiplicative inverse of [[1, 4], [0, 1]].	**[[1, -4], [0, 1]]**
4	Find the multiplicative inverse of [[4, 1], [1, 0]].	**[[0, 1], [1, -4]]**
5	Does [[1, 4], [-1, -4]] have a multiplicative inverse? Use its determinant to explain.	**No. Its determinant is 1(-4) - 4(-1) = 0.**
6	Find both inverses of [[-1, 0], [0, 4]].	**Additive inverse: [[1, 0], [0, -4]]. Multiplicative inverse: [[-1, 0], [0, 1/4]].**

PART 2 · APPLY IT

1. **Inverse: [[1, -1], [0, 1]]. Original values: [8, 6].** The matrix has determinant 1. Multiply its inverse by [14, 6] to get [14 - 6, 6] = [8, 6].
2. **Inverse: [[3, -1/2], [-2, 1/2]]. The booth sold 6 student tickets and 4 adult tickets.** The determinant is 1(6) - 1(4) = 2. Multiply the inverse by [10, 48] to get [30 - 24, -20 + 24] = [6, 4].

CHALLENGE

x = 16. The determinant is 4(4) - x(1) = 16 - x, which is 0 when x = 16, so there is no multiplicative inverse. The additive inverse is [[-4, -16], [-1, -4]] because every matrix has an additive inverse.

Accept an equivalent factored form for a multiplicative inverse. For written explanations, look for a zero determinant as the reason a multiplicative inverse does not exist.