LESSON MAKER PRO

Can You Undo It?

Use determinants to test whether a matrix has an inverse.

HIGH SCHOOL · A2S.NQ.B.5.b

A2S.NQ.B.5.b "Explain the role of the determinant in determining if a square matrix has a multiplicative inverse."

NAME _____ DATE _____

THE INVERSE TEST

For a 2 by 2 matrix [[a, b], [c, d]], the determinant is ad - bc. A square matrix has a multiplicative inverse exactly when its determinant is not 0.

WORKED EXAMPLE

Does [[11, 13], [17, 19]] have a multiplicative inverse?

1. Multiply along the diagonals: 11 × 19 = 209 and 13 × 17 = 221.
2. Subtract: det = 209 - 221 = -12.
3. Since -12 is not 0, the matrix has a multiplicative inverse.

Answer: The determinant is -12, so the matrix has an inverse.

PRACTICE 6 problems

1 Find the determinant of [[2, 1], [3, 2]]. Use it to explain whether the matrix has an inverse.

ANSWER _____

2 Find the determinant of [[2, 3], [4, 6]]. Use it to explain whether the matrix has an inverse.

ANSWER _____

3 Find the determinant of [[0, 2], [-3, 1]]. Use it to explain whether the matrix has an inverse.

ANSWER _____

4 Find the determinant of [[-2, 3], [1, 2]]. Use it to explain whether the matrix has an inverse.

ANSWER _____

5 Find the determinant of [[-3, 2], [6, -4]]. Use it to explain whether the matrix has an inverse.

ANSWER _____

6 For what value of x does [[x, 2], [3, 6]] have no inverse? Show the determinant equation.

ANSWER _____

APPLY IT Show your work

1 A game uses a matrix to turn two joystick inputs into two on-screen movements. The matrix is [[2, 1], [1, 3]]. Can the game use the movements to recover the two inputs uniquely? Use the determinant to explain.

WORK SPACE

ANSWER _____

2 A video editing app turns two slider settings into two color readings using [[1, 2], [2, 4]]. Can the app use the readings to recover the slider settings uniquely? Use the determinant to explain.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For which values of x does [[x, 2], [3, x - 1]] have an inverse? Explain using the determinant.

WORK SPACE AND ANSWER

ANSWER KEY

Can You Undo It?

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the determinant of [[2, 1], [3, 2]]. Use it to explain whether the matrix has an inverse.	**det = 2(2) - 1(3) = 1. It has an inverse because the determinant is not 0.**
2	Find the determinant of [[2, 3], [4, 6]]. Use it to explain whether the matrix has an inverse.	**det = 2(6) - 3(4) = 0. It has no inverse because the determinant is 0.**
3	Find the determinant of [[0, 2], [-3, 1]]. Use it to explain whether the matrix has an inverse.	**det = 0(1) - 2(-3) = 6. It has an inverse because the determinant is not 0.**
4	Find the determinant of [[-2, 3], [1, 2]]. Use it to explain whether the matrix has an inverse.	**det = (-2)(2) - 3(1) = -7. It has an inverse because the determinant is not 0.**
5	Find the determinant of [[-3, 2], [6, -4]]. Use it to explain whether the matrix has an inverse.	**det = (-3)(-4) - 2(6) = 0. It has no inverse because the determinant is 0.**
6	For what value of x does [[x, 2], [3, 6]] have no inverse? Show the determinant equation.	**Set det = 6x - 6 = 0. The matrix has no inverse when x = 1.**

PART 2 · APPLY IT

1. **Yes. The determinant is 2(3) - 1(1) = 5, so the matrix has an inverse.** A nonzero determinant means the transformation is invertible, so each pair of movements has one pair of inputs.
2. **No. The determinant is 1(4) - 2(2) = 0, so the matrix has no inverse.** A zero determinant means the readings cannot determine a unique pair of slider settings.

CHALLENGE

det = x(x - 1) - 6 = (x - 3)(x + 2). The matrix has an inverse when x is not 3 and x is not -2, because its determinant is then nonzero.

Accept equivalent determinant calculations. Explanations must connect a nonzero determinant to the existence of an inverse, or a zero determinant to its absence.