LESSON MAKER PRO

Crowded Categories

Find what a group must contain.

HIGH SCHOOL · FM.AC.A.10

FM.AC.A.10 "Use the Pigeonhole Principle to solve counting problems."

NAME _____ DATE _____

THE PIGEONHOLE PRINCIPLE

When you sort items into groups, at least one group has at least the total number of items divided by the number of groups, rounded up. To guarantee a certain number in one group, first find how many items fit if every group has one fewer, then add 1.

WORKED EXAMPLE

A student puts 46 tracks into 9 playlists. Each track goes into exactly one playlist. What is the least number of tracks that must be in one playlist?

1. If each playlist had at most 5 tracks, the playlists could hold at most 9 × 5 = 45 tracks.
2. There are 46 tracks, so at least one playlist must have more than 5.

Answer: 6 tracks

PRACTICE 6 problems

1 You put 19 cards into 3 sleeves. Each card goes into exactly one sleeve. What is the least number of cards that must be in one sleeve?

ANSWER _____

2 You sort 26 photos into 6 folders. Each photo goes into exactly one folder. What is the least number of photos that must be in one folder?

ANSWER _____

3 Papers are placed on 4 desks. What is the fewest papers needed to guarantee that one desk has at least 6 papers?

ANSWER _____

4 You assign 37 messages to 7 channels. Each message goes into exactly one channel. What is the least number of messages that must be in one channel?

ANSWER _____

5 A manager assigns 22 people to 4 shifts. Each person works exactly one shift. Can every shift have at most 5 people? Explain briefly.

ANSWER _____

6 A bag has 12 marbles of each of 6 colors. How many marbles must you draw without looking to guarantee at least 3 of one color?

ANSWER _____

APPLY IT Show your work

1 At a concert merch pickup, 74 students each choose exactly one of 12 pickup windows. What is the least number of students that must choose the same window?

WORK SPACE

ANSWER _____

2 An esports club lets each player sign up for exactly one of 6 tournament brackets. What is the fewest players who must sign up to guarantee at least 7 players in one bracket?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Each club member records a birth month and chooses exactly one of 3 favorite music genres. What is the fewest members needed to guarantee that two share both a birth month and a favorite genre? Explain why fewer members would not guarantee a match.

WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	You put 19 cards into 3 sleeves. Each card goes into exactly one sleeve. What is the least number of cards that must be in one sleeve?	**7 cards**
2	You sort 26 photos into 6 folders. Each photo goes into exactly one folder. What is the least number of photos that must be in one folder?	**5 photos**
3	Papers are placed on 4 desks. What is the fewest papers needed to guarantee that one desk has at least 6 papers?	**21 papers**
4	You assign 37 messages to 7 channels. Each message goes into exactly one channel. What is the least number of messages that must be in one channel?	**6 messages**
5	A manager assigns 22 people to 4 shifts. Each person works exactly one shift. Can every shift have at most 5 people? Explain briefly.	**No. Four shifts with at most 5 people each can hold only 20 people.**
6	A bag has 12 marbles of each of 6 colors. How many marbles must you draw without looking to guarantee at least 3 of one color?	**13 marbles**

PART 2 · APPLY IT

1. **7 students** If each window had at most 6 students, the windows could serve only 72 students. Since 74 students choose windows, at least one window has 7 or more.
2. **37 players** With at most 6 players per bracket, 6 brackets can hold 36 players. One more player guarantees a bracket with at least 7.

CHALLENGE

37 members. There are 12 × 3 = 36 possible month-and-genre pairs. With 36 members, each could have a different pair. A 37th member must share a pair with someone.

Accept any correct pigeonhole argument. For guarantee questions, a complete explanation can show the largest number of items possible without reaching the target.