LESSON MAKER PRO

Votes and Surprises

Compare winners when the counting rule changes.

HIGH SCHOOL · FM.FD.A.24

FM.FD.A.24 "Identify issues of fairness for different methods of determining a winner using a preferential voting ballot and other voting systems and identify paradoxes that can result."

NAME _____ DATE _____

HOW VOTES ARE COUNTED

Plurality elects the candidate with the most first-choice votes, even without a majority. In instant runoff, eliminate the candidate with the fewest first-choice votes and move those ballots to their next remaining choice until someone has a majority. In a head-to-head race, each ballot counts for whichever of the two candidates is ranked higher; these results can even form a cycle.

WORKED EXAMPLE

The table shows ranked ballots. Who wins by plurality? Who wins by instant runoff?

SAMPLE BALLOT

Voters	1st	2nd	3rd
13	Red	Blue	Gold
9	Blue	Gold	Red
7	Gold	Blue	Red

1. First-choice totals: Red 13, Blue 9, Gold 7. Red wins by plurality.
2. Eliminate Gold, which has the fewest first-choice votes.
3. Move Gold's 7 votes to Blue, their next remaining choice. Blue has 9 + 7 = 16 votes; Red has 13.

Answer: Red wins by plurality. Blue wins by instant runoff, 16 to 13.

PRACTICE 6 problems

1 Who wins by plurality? Does the winner have more than half the votes?

BALLOT 1

Voters	1st	2nd	3rd
4	A	C	B
6	B	C	A
1	C	B	A

ANSWER _____

2 Who wins by instant runoff? Give the final vote totals.

BALLOT 2

Voters	1st	2nd	3rd
8	A	C	B
6	B	C	A
4	C	B	A

ANSWER _____

3 Find the plurality winner and the instant-runoff winner.

BALLOT 3

Voters	1st	2nd	3rd
10	A	B	C
8	B	C	A
6	C	B	A

ANSWER _____

4 In a head-to-head race between A and B, who wins? Give both vote totals.

BALLOT 4

Voters	1st	2nd	3rd
8	A	B	C
6	B	C	A
4	C	A	B

ANSWER _____

5 Does B beat both other candidates head-to-head? Give the totals for B against A and B against C.

BALLOT 5

Voters	1st	2nd	3rd
11	A	B	C
8	B	C	A
6	C	B	A

ANSWER _____

6 Use this points rule: 2 for first choice, 1 for second, and 0 for third. Who wins, and how many points does the winner earn?

BALLOT 6

Voters	1st	2nd	3rd
12	A	B	C
8	B	C	A
6	C	B	A

ANSWER _____

APPLY IT Show your work

1 Students rank three choices for a school festival meal. Which meal wins by plurality, and which wins by instant runoff? Name one reason the results differ.

FESTIVAL MEAL BALLOT

Voters	1st	2nd	3rd
16	Pizza	Tacos	Bowls
12	Tacos	Bowls	Pizza
10	Bowls	Tacos	Pizza

ANSWER _____

2 A film club ranks genres for its next screening. Which genre wins by plurality? Then award 2 points for first choice, 1 for second, and 0 for third. Which genre wins by points, and how many points does it earn?

FILM GENRE BALLOT

Voters	1st	2nd	3rd
14	Action	Comedy	Drama
10	Comedy	Drama	Action
8	Drama	Comedy	Action

ANSWER _____

CHALLENGE One step up

A student council uses the ballot below. Find the winner of each head-to-head race: A against B, B against C, and C against A. Explain why no candidate beats both others.

STUDENT COUNCIL BALLOT

Voters	1st	2nd	3rd
6	A	B	C
5	B	C	A
4	C	A	B

WORK SPACE AND ANSWER

ANSWER KEY

Votes and Surprises

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Who wins by plurality? Does the winner have more than half the votes?	**B wins with 6 votes. Yes, 6 is more than half of 11.**
2	Who wins by instant runoff? Give the final vote totals.	**B wins by instant runoff, 10 votes to A's 8.**
3	Find the plurality winner and the instant-runoff winner.	**A wins by plurality with 10 votes. B wins by instant runoff, 14 to 10.**
4	In a head-to-head race between A and B, who wins? Give both vote totals.	**A wins, 12 votes to B's 6.**
5	Does B beat both other candidates head-to-head? Give the totals for B against A and B against C.	**Yes. B beats A, 14 to 11, and beats C, 19 to 6.**
6	Use this points rule: 2 for first choice, 1 for second, and 0 for third. Who wins, and how many points does the winner earn?	**B wins with 34 points.**

PART 2 · APPLY IT

1. **Pizza wins by plurality. Tacos wins by instant runoff because the 10 Bowls voters rank Tacos above Pizza.** Pizza has the most first-choice votes, 16. Eliminate Bowls and transfer its 10 votes to Tacos, giving Tacos 22 votes to Pizza's 16.
2. **Action wins by plurality. Comedy wins by points with 42.** Action has the most first-choice votes, 14. Comedy earns 14 points for second choice on the Action ballots, 20 for first choice on its own ballots, and 8 for second choice on the Drama ballots.

CHALLENGE

A beats B, 10 to 5; B beats C, 11 to 4; C beats A, 9 to 6. Each candidate loses one head-to-head race, so none beats both others.

Accept equivalent explanations of why the festival results differ. In the challenge, the three results form a head-to-head cycle.