LESSON MAKER PRO

Vote Count Switch

See how a voting rule can change the winner.

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FM.FD.A "Various methods for determining a winner in a voting system can result in paradoxes or other issues of fairness."

NAME _____ DATE _____

TWO WAYS TO WIN

In a plurality vote, the choice with the most first-choice votes wins. In an instant runoff with three choices, remove the choice with the fewest first-choice votes. Move those ballots to the voters' second choices, then compare the two remaining totals.

WORKED EXAMPLE

Students rank three after-school activities. Find the plurality winner and the instant-runoff winner.

ACTIVITY RANKINGS

Voters	1st	2nd	3rd
16	Skate	Music	Art
11	Music	Art	Skate
7	Art	Music	Skate

1. First-choice totals: Skate 16, Music 11, Art 7. Skate wins by plurality.
2. Remove Art, which has the fewest first-choice votes.
3. The 7 Art ballots move to Music. Music has 11 + 7 = 18 votes; Skate has 16.

Answer: Plurality winner: Skate. Instant-runoff winner: Music.

PRACTICE 6 problems

1 Find the plurality winner and the instant-runoff winner.

RANKED VOTES

Voters	1st	2nd	3rd
12	A	C	B
9	B	A	C
5	C	B	A

ANSWER _____

2 After removing the choice with the fewest first-choice votes, how many votes does each remaining choice have?

RANKED VOTES

Voters	1st	2nd	3rd
14	A	B	C
10	B	C	A
4	C	A	B

ANSWER _____

3 Do plurality and instant runoff give the same winner? Name both winners.

RANKED VOTES

Voters	1st	2nd	3rd
13	A	B	C
8	B	A	C
6	C	B	A

ANSWER _____

4 Which choice is removed first in an instant runoff, and which choice wins?

RANKED VOTES

Voters	1st	2nd	3rd
15	A	C	B
12	B	A	C
3	C	A	B

ANSWER _____

5 Find the plurality winner and the instant-runoff winner.

RANKED VOTES

Voters	1st	2nd	3rd
17	A	C	B
14	B	A	C
9	C	B	A

ANSWER _____

6 Do plurality and instant runoff agree? Name the winner and its instant-runoff vote total.

RANKED VOTES

Voters	1st	2nd	3rd
20	A	B	C
18	B	C	A
2	C	A	B

ANSWER _____

APPLY IT Show your work

1 Your school is choosing a theme for a Friday event. Students ranked Games, Concert, and Film. Which theme wins by plurality? Which wins by instant runoff?

EVENT THEME RANKINGS

Voters	1st	2nd	3rd
22	Games	Concert	Film
19	Concert	Film	Games
8	Film	Concert	Games

ANSWER _____

2 A student club is choosing a hoodie design. Members ranked Wave, Star, and Bolt. Which design wins by instant runoff? Did that design have the most first-choice votes?

HOODIE DESIGN RANKINGS

Voters	1st	2nd	3rd
18	Wave	Star	Bolt
14	Star	Wave	Bolt
12	Bolt	Star	Wave

ANSWER _____

CHALLENGE One step up

In a club election, members ranked X, Y, and Z. Compare each pair separately: X against Y, Y against Z, and Z against X. For each pair, count a voter's ballot for the choice ranked higher. Give each score, then explain why these head-to-head results do not identify one choice that beats every other choice.

CLUB ELECTION RANKINGS

Voters	1st	2nd	3rd
5	X	Y	Z
4	Y	Z	X
3	Z	X	Y

WORK SPACE AND ANSWER

ANSWER KEY

Vote Count Switch

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the plurality winner and the instant-runoff winner.	**Plurality: A. Instant runoff: B.**
2	After removing the choice with the fewest first-choice votes, how many votes does each remaining choice have?	**A: 18 votes. B: 10 votes.**
3	Do plurality and instant runoff give the same winner? Name both winners.	**No. Plurality: A. Instant runoff: B.**
4	Which choice is removed first in an instant runoff, and which choice wins?	**C is removed. A wins with 18 votes.**
5	Find the plurality winner and the instant-runoff winner.	**Plurality: A. Instant runoff: B.**
6	Do plurality and instant runoff agree? Name the winner and its instant-runoff vote total.	**Yes. A wins under both methods and has 22 instant-runoff votes.**

PART 2 · APPLY IT

1. **Plurality: Games. Instant runoff: Concert.** Games has the most first-choice votes, 22. Remove Film and move its 8 ballots to Concert, giving Concert 27 votes to Games's 22.
2. **Star wins the instant runoff with 26 votes. No, Wave had the most first-choice votes, 18.** Remove Bolt and move its 12 ballots to Star. Star finishes with 14 + 12 = 26 votes, while Wave stays at 18.

CHALLENGE

X beats Y, 8 to 4. Y beats Z, 9 to 3. Z beats X, 7 to 5. Each choice loses one head-to-head contest, so none beats every other choice.

Accept either order when students name two winners, as long as each method is labeled. In the challenge, accept any explanation that correctly identifies the cycle of head-to-head wins.