LESSON MAKER PRO

Who Gets What?

Use personal values to test whether a division is fair.

HIGH SCHOOL · FM.FDV.A

FM.FDV.A "Methods used to solve non-trivial problems of division of objects often reveal issues of fairness."

NAME _____ DATE _____

TWO TESTS FOR FAIRNESS

Add a person's values for all the objects, then divide by the number of people to find that person's fair share. A division is proportional if everyone gets at least their own fair share. A division is envy-free if no one values another person's bundle more than their own.

WORKED EXAMPLE

Ari gets the pin, and Bea gets the poster. Is this division proportional and envy-free?

PERSONAL VALUES

Person	Poster	Pin
Ari	11	17
Bea	23	13

1. Ari's fair share is (11 + 17)/2 = 14. Bea's is (23 + 13)/2 = 18.
2. Ari gets a value of 17, and Bea gets a value of 23. Both meet their fair shares.
3. Ari values Bea's poster at 11, less than 17. Bea values Ari's pin at 13, less than 23.

Answer: The division is proportional and envy-free.

PRACTICE 6 problems

1 Ada and Ben will divide objects X and Y. What is each person's fair share?

PERSONAL VALUES

Person	X	Y
Ada	6	4
Ben	2	8

ANSWER _____

2 Mira gets Y, and Sol gets X. Is the division proportional?

PERSONAL VALUES

Person	X	Y
Mira	3	9
Sol	10	2

ANSWER _____

3 Lia gets A. Theo gets B and C. Who falls short of their fair share, and by how much?

PERSONAL VALUES

Person	A	B	C
Lia	8	4	6
Theo	2	9	5

ANSWER _____

4 Ava gets A, Bo gets B, and Cy gets C. Who envies whom?

PERSONAL VALUES

Person	A	B	C
Ava	5	8	2
Bo	3	7	4
Cy	4	2	9

ANSWER _____

5 Rowan gets A. Quinn gets B and C. Move one object from Quinn to Rowan so both receive at least their fair share. Which object moves?

PERSONAL VALUES

Person	A	B	C
Rowan	7	5	4
Quinn	2	8	6

ANSWER _____

6 Three people divide two objects. Each person values both objects above zero. If the objects cannot be split or shared, can everyone receive at least their fair share? Explain.

ANSWER _____

APPLY IT Show your work

1 Nia and Omar win three concert merch items. They consider two plans: Plan A gives Nia the poster and pin and Omar the hoodie. Plan B gives Nia the hoodie and Omar the poster and pin. Which plan is proportional? Explain using their personal values.

VALUE OF CONCERT MERCH

Person	Poster	Hoodie	Pin
Nia	6	9	3
Omar	8	4	6

ANSWER _____

2 Three friends split passes to different festival events. Imani gets the music pass, Jules gets the film pass, and Kai gets the food pass. Is the division proportional? Is it envy-free? Explain.

VALUE OF FESTIVAL PASSES

Person	Music	Film	Food
Imani	8	4	3
Jules	3	7	5
Kai	4	6	5

ANSWER _____

CHALLENGE One step up

Three friends divide four prizes. Each object goes to one person, and a person may get more than one. Find a division that is both proportional and envy-free. Show each person's value for their own bundle and explain why nobody envies another bundle.

PERSONAL VALUES

Person	X	Y	Z	W
A	6	5	2	1
B	2	3	7	4
C	1	4	5	8

WORK SPACE AND ANSWER

Who Gets What?

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Ada and Ben will divide objects X and Y. What is each person's fair share?	**Ada: 5; Ben: 5**
2	Mira gets Y, and Sol gets X. Is the division proportional?	**Yes. Mira gets 9 and Sol gets 10; each person's fair share is 6.**
3	Lia gets A. Theo gets B and C. Who falls short of their fair share, and by how much?	**Lia falls short by 1.**
4	Ava gets A, Bo gets B, and Cy gets C. Who envies whom?	**Ava envies Bo.**
5	Rowan gets A. Quinn gets B and C. Move one object from Quinn to Rowan so both receive at least their fair share. Which object moves?	**C moves to Rowan.**
6	Three people divide two objects. Each person values both objects above zero. If the objects cannot be split or shared, can everyone receive at least their fair share? Explain.	**No. At least one person receives no object, so they get zero value, less than their positive fair share.**

PART 2 · APPLY IT

1. **Plan B only. Nia gets 9 and Omar gets 14, meeting their fair shares of 9 each.** In Plan A, Omar gets a value of 4, below his fair share of 9. In Plan B, both people meet their fair shares.
2. **It is proportional but not envy-free. Everyone gets at least their fair share of 5, but Kai values Jules's film pass at 6, more than Kai's own food pass at 5.** Each person's total value is 15, so each fair share is 5. Their assigned values are 8, 7, and 5, but Kai envies Jules.

CHALLENGE

Give X and Y to A, Z to B, and W to C. Their own bundle values are 11, 7, and 8, above their fair shares of 14/3, 16/3, and 6. A values the other bundles at 2 and 1; B values them at 5 and 4; C values them at 5 and 5. Nobody envies another bundle.

Accept equivalent fractions or decimals for fair shares. For envy-free explanations, students must compare each person's own bundle with the other bundles using that same person's values.