LESSON MAKER PRO

Secret Matrix Messages

Encode letter pairs, then reverse the code.

HIGH SCHOOL · FM.IP.A.30.b

FM.IP.A.30.b "Use matrices and their inverses to encode and decode messages."

NAME _____ DATE _____

HOW THE CODE WORKS

Let A = 1, B = 2, and so on through Z = 26. Write two letters as a column pair (u,v), then multiply by a 2 × 2 matrix to encode them. To decode a coded pair, multiply it by the matrix's inverse.

WORKED EXAMPLE

Use $E = [[5,2],[7,3]]$ to encode HI. Then use E^{-1} to check your result.

1. HI becomes the column pair (8,9).
2. $E(8,9) = (5 \times 8 + 2 \times 9, 7 \times 8 + 3 \times 9) = (58,83)$.
3. The determinant is $5 \times 3 - 2 \times 7 = 1$, so $E^{-1} = [[3,-2],[-7,5]]$.
4. $E^{-1}(58,83) = (3 \times 58 - 2 \times 83, -7 \times 58 + 5 \times 83) = (8,9)$.

Answer: HI encodes as (58,83). The inverse returns (8,9), which is HI.

PRACTICE 6 problems

1 Find the inverse of M = [[1,1],[1,2]].

ANSWER _____

2 Use M = [[1,1],[1,2]] to encode GO.

ANSWER _____

3 Use M = [[1,1],[1,2]] to encode UP.

ANSWER _____

4 Use the inverse of M = [[1,1],[1,2]] to decode (26,37).

ANSWER _____

5 Use the inverse of M = [[1,1],[1,2]] to decode (18,23).

ANSWER _____

6 Each coded pair stands for two letters. Use the inverse of M = [[1,1],[1,2]] to decode (29,44), (25,30) in order.

ANSWER _____

APPLY IT Show your work

1 Your school music club wants to hide the artist tag DJ on a digital ticket. Encode DJ using M = [[1,1],[1,2]]. What pair should appear on the ticket?

WORK SPACE

ANSWER _____

2 A game night group sends a two-block clue encoded with M = [[1,1],[1,2]]: (21,30), (27,32). Decode the blocks in order. What word tells players how the event will be broadcast?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A different code uses N = [[2,1],[1,1]]. Decode (45,25), (15,14) in order. Then explain in one sentence why multiplying by N^{-1} recovers the original letter pairs.

WORK SPACE AND ANSWER

Secret Matrix Messages

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the inverse of M = [[1,1],[1,2]].	**M^{-1} = [[2,-1],[-1,1]]**
2	Use M = [[1,1],[1,2]] to encode GO.	**(22,37)**
3	Use M = [[1,1],[1,2]] to encode UP.	**(37,53)**
4	Use the inverse of M = [[1,1],[1,2]] to decode (26,37).	**OK**
5	Use the inverse of M = [[1,1],[1,2]] to decode (18,23).	**ME**
6	Each coded pair stands for two letters. Use the inverse of M = [[1,1],[1,2]] to decode (29,44), (25,30) in order.	**NOTE**

PART 2 · APPLY IT

1. **(14,24)** DJ becomes (4,10). Multiplying by M gives (4 + 10, 4 + 2 × 10) = (14,24).
2. **LIVE** Multiplying each block by M^{-1} = [[2,-1],[-1,1]] gives (12,9) and (22,5). These pairs spell LI and VE.

CHALLENGE

TEAM. Because $N^{-1}N$ is the identity matrix, multiplying each coded pair by N^{-1} restores its original letter pair.

Treat each ordered pair as a column when multiplying. For the challenge, N^{-1} = [[1,-1],[-1,2]], which gives (20,5) and (1,13).