LESSON MAKER PRO

Crack the Matrix

Encode and decode pairs using a matrix key.

HIGH SCHOOL · FM.IP.A.30

FM.IP.A.30 "Apply ciphers (encryption and decryption algorithms) and cryptosystems for encrypting and decrypting including symmetric-key or public-key systems. a. Use modular arithmetic to apply RSA (Rivest-Shamir-Adleman) public-key cryptosystems. b..."

NAME _____ DATE _____

THE BIG IDEA

A message can use A = 1, B = 2, and so on through Z = 26. To encode a row pair (a,b), multiply it by a key matrix K: if K has rows (p,q) and (r,s), then (a,b)K = (ap + br, aq + bs). To decode, multiply the cipher pair by K^{-1}, which exists only when ps - qr is not 0.

WORKED EXAMPLE

The figure shows key matrix K. Encode (5,9), then use the inverse to check your result.

KEY MATRIX K

Column 1	Column 2
2	1
3	2

1. Encode: (5,9)K = (5(2) + 9(3), 5(1) + 9(2)) = (37,23).
2. Find the determinant: 2(2) - 1(3) = 1.
3. Since the determinant is 1, K^{-1} has rows (2,-1) and (-3,2).
4. Check: (37,23)K^{-1} = (74 - 69, -37 + 46) = (5,9).

Answer: The cipher pair is (37,23), and decoding returns (5,9).

PRACTICE 6 problems

1 The figure shows key matrix K. Encode (3,4).

KEY MATRIX K

Column 1	Column 2
1	1
1	2

ANSWER _____

2 Key matrix K has first row (2,1) and second row (1,1). Encode (4,2).

ANSWER _____

3 The figure shows key matrix K. Encode (8,5).

KEY MATRIX K

Column 1	Column 2
1	2
0	1

ANSWER _____

4 Key matrix K has first row (1,1) and second row (2,3). Decode the cipher pair (12,17).

ANSWER _____

5 The figure shows key matrix K. Decode the cipher pair (14,24). Give the number pair and its two letters using A = 1 through Z = 26.

KEY MATRIX K

Column 1	Column 2
2	3
1	2

ANSWER _____

6 Key matrix K has first row (1,0) and second row (2,1). Decode the cipher pair (15,3).

ANSWER _____

APPLY IT Show your work

1 A concert crew sends the initials DJ as a coded pair. Using A = 1 through Z = 26, they turn DJ into (4,10). Their key matrix has first row (1,2) and second row (1,3). What cipher pair do they send?

WORK SPACE

ANSWER _____

2 A school club sends a two-letter pickup signal as the cipher pair (29,22). Its key matrix has first row (2,1) and second row (1,1). Decode the pair, then use A = 1 through Z = 26 to find the signal.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student claims the key matrix in the figure can decode every cipher pair because none of its entries is 0. Explain why the claim is false, and give two different pairs of positive whole numbers that this key encodes as the same cipher pair.

KEY MATRIX K

Column 1	Column 2
2	4
1	2

WORK SPACE AND ANSWER

Crack the Matrix

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The figure shows key matrix K. Encode (3,4).	**(7,11)**
2	Key matrix K has first row (2,1) and second row (1,1). Encode (4,2).	**(10,6)**
3	The figure shows key matrix K. Encode (8,5).	**(8,21)**
4	Key matrix K has first row (1,1) and second row (2,3). Decode the cipher pair (12,17).	**(2,5)**
5	The figure shows key matrix K. Decode the cipher pair (14,24). Give the number pair and its two letters using A = 1 through Z = 26.	**(4,6); DF**
6	Key matrix K has first row (1,0) and second row (2,1). Decode the cipher pair (15,3).	**(9,3)**

PART 2 · APPLY IT

1. **(14,38)** Multiply the row pair by the key: (4(1) + 10(1), 4(2) + 10(3)) = (14,38). Cipher values can be greater than 26.
2. **(7,15); GO** The key's inverse has rows (1,-1) and (-1,2). Multiplying (29,22) by that inverse gives (7,15), or GO.

CHALLENGE

The determinant is 2(2) - 4(1) = 0, so the key has no inverse. Both (1,4) and (2,2) encode as (6,12), so that cipher pair cannot be decoded uniquely.

Accept correct pairs shown with or without parentheses. For the challenge, accept any two distinct positive whole-number pairs that produce the same cipher pair, along with a valid explanation that the key cannot decode uniquely.