LESSON MAKER PRO

Always or Never

Test whether a logical statement can ever change its truth value.

HIGH SCHOOL · FM.LR.A.4.a

FM.LR.A.4.a "Determine whether a logical argument is a tautology or a contradiction."

NAME _____ DATE _____

THE TRUTH TEST

A tautology is true for every possible assignment of truth values. A contradiction is false for every possible assignment. Check all assignments: ¬ means not, ∧ means and, ∨ means or, → means if...then, and ↔ means if and only if. A statement p → q is false only when p is true and q is false; p ↔ q is true when p and q have the same truth value.

WORKED EXAMPLE

Is r ∨ ¬r a tautology or a contradiction?

1. If r is true, ¬r is false, so r ∨ ¬r is true.
2. If r is false, ¬r is true, so r ∨ ¬r is true.

Answer: Tautology. The statement is true in both cases.

PRACTICE 6 problems

1 Decide whether ¬(p ∧ ¬p) is a tautology or a contradiction.

ANSWER _____

2 Decide whether p ∧ ¬(p ∨ q) is a tautology or a contradiction.

ANSWER _____

3 Decide whether (p → q) ∨ (q → p) is a tautology or a contradiction.

ANSWER _____

4 Decide whether (p ↔ q) ∧ p ∧ ¬q is a tautology or a contradiction.

ANSWER _____

5 Decide whether (p ∧ q) → p is a tautology or a contradiction.

ANSWER _____

6 Decide whether (p ∨ q) ∧ ¬p ∧ ¬q is a tautology or a contradiction.

ANSWER _____

APPLY IT Show your work

1 For a livestream, let p mean the stream is live and q mean chat is open. A post claims, "Chat is open, or the stream is live, or neither chat nor the stream is available." Write the claim using p and q. Is it a tautology or a contradiction?

WORK SPACE

ANSWER _____

2 For a school election, let p mean voting is open and q mean the voting link works. A post claims, "If voting is open, the link works. Voting is open, but the link does not work." Write the combined claim using p and q. Is it a tautology or a contradiction?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Decide whether $((p \rightarrow q) \wedge (q \rightarrow r) \wedge p) \rightarrow r$ is a tautology or a contradiction. Explain why your answer holds for every assignment of truth values.

WORK SPACE AND ANSWER

ANSWER KEY # Always or Never

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Decide whether ¬(p ∧ ¬p) is a tautology or a contradiction.	**Tautology**
2	Decide whether p ∧ ¬(p ∨ q) is a tautology or a contradiction.	**Contradiction**
3	Decide whether (p → q) ∨ (q → p) is a tautology or a contradiction.	**Tautology**
4	Decide whether (p ↔ q) ∧ p ∧ ¬q is a tautology or a contradiction.	**Contradiction**
5	Decide whether (p ∧ q) → p is a tautology or a contradiction.	**Tautology**
6	Decide whether (p ∨ q) ∧ ¬p ∧ ¬q is a tautology or a contradiction.	**Contradiction**

PART 2 · APPLY IT

1. **q ∨ p ∨ (¬q ∧ ¬p); tautology** If either chat is open or the stream is live, the first part is true. If neither is true, ¬q ∧ ¬p is true.
2. **(p → q) ∧ p ∧ ¬q; contradiction** The last two parts require p to be true and q to be false. Those truth values make p → q false, so the combined claim can never be true.

CHALLENGE

Tautology. If the expression before the final → is true, p is true, so q must be true and then r must be true. If that expression is false, the full conditional is true.

Accept equivalent expressions for the two word problems. For the challenge, accept a complete truth table or an explanation covering both cases of the final conditional.