LESSON MAKER PRO

Logic Under Test

Check truth values and test conclusions.

HIGH SCHOOL · FM.LR.A.4

FM.LR.A.4 "Determine whether a logical argument is valid or invalid, using laws of logic such as the law of syllogism and the law of detachment. a. Determine whether a logical argument is a tautology or a contradiction."

NAME _____ DATE _____

TEST EVERY CASE

A conditional p → q is false only when p is true and q is false. A statement is a tautology if it is true in every case and a contradiction if it is false in every case. An argument is valid if there is no case in which all its premises are true and its conclusion is false.

WORKED EXAMPLE

Classify (r AND s) → r as a tautology, a contradiction, or neither.

1. List the cases for (r, s): (T, T), (T, F), (F, T), (F, F).
2. In those cases, r AND s is T, F, F, F.
3. The conditional is T in every case: T → T, F → T, F → F, F → F.

Answer: Tautology

PRACTICE 6 problems

1 Classify p OR NOT p as a tautology, a contradiction, or neither.

ANSWER _____

2 Classify p AND NOT p as a tautology, a contradiction, or neither.

ANSWER _____

3 Classify p → q as a tautology, a contradiction, or neither.

ANSWER _____

4 Is this argument valid or invalid? p → q. p. Therefore, q.

ANSWER _____

5 Is this argument valid or invalid? p → q. NOT p. Therefore, NOT q.

ANSWER _____

6 Is this argument valid or invalid? p → q. q → t. Therefore, p → t.

ANSWER _____

APPLY IT Show your work

1 At a concert, anyone with a VIP ticket may enter early. Jamie may enter early. Is the conclusion that Jamie has a VIP ticket valid? Explain with a possible alternative.

WORK SPACE

ANSWER _____

2 If the esports team wins its qualifier, it advances to the city finals. If it advances to the city finals, it earns a travel stipend. The team wins its qualifier. Is the conclusion that it earns a travel stipend valid?

WORK SPACE

ANSWER _____

CHALLENGE One step up

An argument has these premises: p → q, q → t, and NOT t. Its conclusion is NOT p. Is the argument valid? Is the statement [(p → q) AND (q → t) AND NOT t] → NOT p a tautology? Explain.

WORK SPACE AND ANSWER

Logic Under Test

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Classify p OR NOT p as a tautology, a contradiction, or neither.	**Tautology**
2	Classify p AND NOT p as a tautology, a contradiction, or neither.	**Contradiction**
3	Classify $p \rightarrow q$ as a tautology, a contradiction, or neither.	**Neither**
4	Is this argument valid or invalid? $p \rightarrow q$. p. Therefore, q.	**Valid**
5	Is this argument valid or invalid? $p \rightarrow q$. NOT p. Therefore, NOT q.	**Invalid**
6	Is this argument valid or invalid? $p \rightarrow q$. $q \rightarrow t$. Therefore, $p \rightarrow t$.	**Valid**

PART 2 · APPLY IT

1. **Invalid. Jamie could have a staff pass that also allows early entry.** Early entry does not establish that Jamie has a VIP ticket. The premise could be true while the conclusion is false.
2. **Valid** The law of syllogism links winning the qualifier to earning a stipend. The law of detachment then applies because the team won its qualifier.

CHALLENGE

The argument is valid, and the statement is a tautology. If p were true, q and then t would be true, contradicting NOT t. Thus, whenever all the premises are true, NOT p is true.

For invalid arguments, accept any case that makes the premises true and the conclusion false. For the challenge, accept a truth-table explanation or equivalent reasoning.