LESSON MAKER PRO

Flip the Claim

Prove each claim by proving its contrapositive. All variables represent integers.

HIGH SCHOOL · FM.LR.A.5

FM.LR.A.5 "Prove a statement indirectly by proving the contrapositive of the statement."

NAME _____ DATE _____

REVERSE AND NEGATE

To prove a statement of the form "If P, then Q," you can prove "If not Q, then not P." Reverse the two parts, negate each one, and then prove the new statement.

WORKED EXAMPLE

Prove: If $n + 11 > 19$, then $n > 8$.

1. Write the contrapositive: If $n \leq 8$, then $n + 11 \leq 19$.
2. Assume $n \leq 8$. Add 11 to both sides to get $n + 11 \leq 19$.
3. The contrapositive is true, so the original claim is true.

Answer: If $n \leq 8$, then $n + 11 \leq 19$, as shown by adding 11 to both sides of $n \leq 8$.

PRACTICE 6 problems

1 Prove by contrapositive: If $3n + 7 > 22$, then $n > 5$.

ANSWER _____

2 Prove by contrapositive: If n^2 is even, then n is even.

ANSWER _____

3 Prove by contrapositive: If n^2 is divisible by 3, then n is divisible by 3.

ANSWER _____

4 Prove by contrapositive: If $7n + 4$ is odd, then n is odd.

ANSWER _____

5 Prove by contrapositive: If $n + 9$ is divisible by 3, then n is divisible by 3.

ANSWER _____

6 Prove by contrapositive: If $3n + 6$ is divisible by 4, then n is even.

ANSWER _____

APPLY IT Show your work

1 A streaming plan costs $4 plus $3 for each movie rental. Prove by contrapositive: If the total cost is an odd number of dollars, then the number of rentals is odd.

WORK SPACE

ANSWER _____

2 A game team earns 2 points per win and 1 point per puzzle solved. Prove by contrapositive: If the team's total points are odd, then the number of puzzles solved is odd.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Prove by contrapositive: If n^2 is divisible by 16, then n is divisible by 4. Explain why your proof covers every integer not divisible by 4.

WORK SPACE AND ANSWER

Flip the Claim

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Prove by contrapositive: If $3n + 7 > 22$, then $n > 5$.	**If $n \leq 5$, then $3n + 7 \leq 3(5) + 7 = 22$. This proves the contrapositive.**
2	Prove by contrapositive: If n^2 is even, then n is even.	**If n is odd, write $n = 2k + 1$. Then $n^2 = 2(2k^2 + 2k) + 1$, which is odd.**
3	Prove by contrapositive: If n^2 is divisible by 3, then n is divisible by 3.	**If n is not divisible by 3, it leaves a remainder of 1 or 2 when divided by 3. In either case, n^2 leaves a remainder of 1, so it is not divisible by 3.**
4	Prove by contrapositive: If $7n + 4$ is odd, then n is odd.	**If n is even, write $n = 2k$. Then $7n + 4 = 14k + 4 = 2(7k + 2)$, which is even.**
5	Prove by contrapositive: If $n + 9$ is divisible by 3, then n is divisible by 3.	**If n is not divisible by 3, it leaves a remainder of 1 or 2 when divided by 3. Adding 9 does not change that remainder, so $n + 9$ is not divisible by 3.**
6	Prove by contrapositive: If $3n + 6$ is divisible by 4, then n is even.	**If n is odd, write $n = 2k + 1$. Then $3n + 6 = 6k + 9$, which is odd and therefore not divisible by 4.**

PART 2 · APPLY IT

1. **If the number of rentals is even, write it as $2k$. The total is $4 + 3(2k) = 2(2 + 3k)$, which is even.** Prove the contrapositive by starting with an even number of rentals. Show that the resulting cost is even.
2. **If the team solves an even number of puzzles, write that number as $2k$. With w wins, the total is $2w + 2k = 2(w + k)$, which is even.** Prove the contrapositive by starting with an even number of puzzles solved. The points from wins are also even, so the total is even.

CHALLENGE

If n is not divisible by 4, then n is odd or $n = 4k + 2$ for some integer k. If n is odd, n^2 is odd, so 16 does not divide it. Otherwise, $n^2 = (4k + 2)^2 = 16(k^2 + k) + 4$, so 16 does not divide it. These cases cover every integer not divisible by 4.

Accept equivalent algebraic or remainder-based proofs. Each response should state or clearly establish the contrapositive, not merely repeat the original claim.