LESSON MAKER PRO

Logic Under Test

Use witnesses and counterexamples to test claims.

HIGH SCHOOL · FM.LR.A

FM.LR.A "The validity of a statement or argument can be determined using the models and language of first order logic."

NAME _____ DATE _____

THE BIG IDEA

In a domain, "every A is B" means no A falls outside B. "Some A is B" needs at least one example that belongs to both groups. An argument is valid if its conclusion must be true whenever its premises are true.

WORKED EXAMPLE

Let U = {14, 18, 22}. Is this statement true for U? "If every number in U is even, then some number in U is divisible by 4."

1. Check the first part: 14, 18, and 22 are all even.
2. Check the second part: none of these numbers is divisible by 4.
3. The first part is true and the second part is false, so the "if, then" statement is false.

Answer: False.

PRACTICE 6 problems

1 Let U = {1, 3, 5}, A = {1, 3}, and B = {3, 5}. Is "Every A is B" true? If not, give a counterexample.

ANSWER _____

2 Let U = {2, 4, 6}, A = {2, 6}, and B = {4, 6}. Is "Some A is B" true? If so, give a witness.

ANSWER _____

3 Let U = {7, 8, 9}, A = {}, and B = {8}. Is "Every A is B" true or false?

ANSWER _____

4 Let U = {10, 11, 12, 13}, A = {10, 12}, and B = {11, 12}. Is "Not every A is B" true? Give a witness if it is true.

ANSWER _____

5 Premises: Every club member is a volunteer. Nia is a club member. Conclusion: Nia is a volunteer. Is the argument valid?

ANSWER _____

6 Premises: Some gamers are artists. Some artists are runners. Conclusion: Some gamers are runners. Is the argument valid? Use two people to give a counterexample if it is not.

ANSWER _____

APPLY IT Show your work

1 At a concert, every backstage-pass holder gets a blue wristband. Every blue-wristband holder can enter the soundcheck. Talia has a backstage pass. Must Talia be allowed into the soundcheck? Explain.

WORK SPACE

ANSWER _____

2 Every student with a verified email can use a school app's new chat feature. Rowan can use the feature. Must Rowan have a verified email? Give a possible situation that supports your answer.

WORK SPACE

ANSWER _____

CHALLENGE One step up

The table shows which songs three students like. "For every student, there is a song that student likes" is true. Does it follow that "There is a song every student likes"? Decide whether the argument is valid, and use the table to explain.

SONGS STUDENTS LIKE

Student	Song A	Song B	Song C
Ava	Yes	No	No
Ben	No	Yes	No
Cy	No	No	Yes

WORK SPACE AND ANSWER

Logic Under Test

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let U = {1, 3, 5}, A = {1, 3}, and B = {3, 5}. Is "Every A is B" true? If not, give a counterexample.	**False; 1 is in A but not in B.**
2	Let U = {2, 4, 6}, A = {2, 6}, and B = {4, 6}. Is "Some A is B" true? If so, give a witness.	**True; 6 is in both A and B.**
3	Let U = {7, 8, 9}, A = {}, and B = {8}. Is "Every A is B" true or false?	**True.**
4	Let U = {10, 11, 12, 13}, A = {10, 12}, and B = {11, 12}. Is "Not every A is B" true? Give a witness if it is true.	**True; 10 is in A but not in B.**
5	Premises: Every club member is a volunteer. Nia is a club member. Conclusion: Nia is a volunteer. Is the argument valid?	**Valid.**
6	Premises: Some gamers are artists. Some artists are runners. Conclusion: Some gamers are runners. Is the argument valid? Use two people to give a counterexample if it is not.	**Invalid. Kai is a gamer and an artist but not a runner; Lee is an artist and a runner but not a gamer.**

PART 2 · APPLY IT

1. **Yes. Talia must be allowed into the soundcheck.** Talia's backstage pass means she gets a blue wristband. Her blue wristband means she can enter the soundcheck.
2. **No. Rowan could have access through a guest invite without having a verified email.** The rule guarantees access for students with verified emails. It does not say they are the only students who can get access.

CHALLENGE

Invalid. Each student likes a song, so the premise is true. No song is liked by all three students, so the conclusion is false.

Accept any countermodel for problem 6 in which both premises hold but the conclusion fails. In problem 3, the universal statement is true because A has no members.