LESSON MAKER PRO

Network Detectives

Count connections and identify graph properties.

HIGH SCHOOL · FM.N.A.16.a

FM.N.A.16.a "Identify properties of simple graphs, complete graphs, bipartite graphs, complete bipartite graphs, and trees."

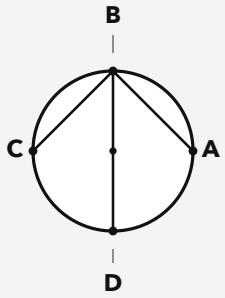
NAME _____ DATE _____

COUNT AND CLASSIFY

In a simple graph, an edge joins two different vertices, and a pair of vertices has at most one edge. A complete graph with n vertices has n(n - 1)/2 edges. A complete bipartite graph has two groups, with every vertex in one group joined to every vertex in the other, so groups of a and b have ab edges. A tree is connected and has no cycles, so a tree with n vertices has n - 1 edges.

WORKED EXAMPLE

Is the graph a tree? How many edges does it have? The curved circle is a guide, not an edge.

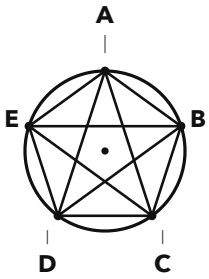


1. Count 4 vertices.
2. Count 3 straight segments, so there are 3 edges.
3. Every vertex is connected, and there is no cycle.
4. The graph is a tree; 3 edges also matches 4 - 1.

Answer: Yes, it is a tree. It has 3 edges.

PRACTICE 6 problems

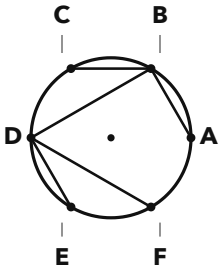
1 The curved circle is a guide, not an edge. Is this graph complete? How many edges does it have?



ANSWER _____

_____

2 The curved circle is a guide, not an edge. Is this graph a tree? How many edges does it have?



ANSWER _____

_____

3 How many edges does a complete graph with 8 vertices have?

ANSWER _____

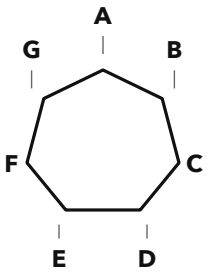
4 How many edges does a complete bipartite graph with groups of 2 and 7 vertices have?

ANSWER _____

5 A tree has 12 vertices. How many edges does it have?

ANSWER _____

6 Is this graph bipartite? Explain briefly.



ANSWER _____

_____

APPLY IT Show your work

1 Nine friends start a group chat. Each pair of friends sends one direct message to each other. How many direct messages are sent?

WORK SPACE

ANSWER _____

2 For an online music event, each of 7 singers records one duet with each of 8 producers. Singers do not record duets with other singers, and producers do not record duets with other producers. How many duets are recorded?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A simple graph has 10 vertices and 45 edges. Could it be a tree? Could it be complete? Explain using edge counts.

WORK SPACE AND ANSWER

Network Detectives

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The curved circle is a guide, not an edge. Is this graph complete? How many edges does it have?	**Yes, it is complete. It has 10 edges.**
2	The curved circle is a guide, not an edge. Is this graph a tree? How many edges does it have?	**Yes, it is a tree. It has 5 edges.**
3	How many edges does a complete graph with 8 vertices have?	**28 edges**
4	How many edges does a complete bipartite graph with groups of 2 and 7 vertices have?	**14 edges**
5	A tree has 12 vertices. How many edges does it have?	**11 edges**
6	Is this graph bipartite? Explain briefly.	**No. The cycle has 7 edges, so alternating between two groups returns to the start with an edge inside one group.**

PART 2 · APPLY IT

1. **36 direct messages** Each pair sends one message. Count the pairs with $9(9 - 1)/2 = 36$.
2. **56 duets** The two groups form a complete bipartite graph. Multiply $7 \times 8 = 56$.

CHALLENGE

It cannot be a tree, which would have 10 - 1 = 9 edges. It must be complete because $10(10 - 1)/2 = 45$ is the greatest possible number of edges for a simple graph with 10 vertices.

For the figures with circles, count only the straight segments as edges. Accept any clear explanation that the odd cycle in the final practice figure prevents a bipartite split.