LESSON MAKER PRO

Network Connections

Count edges and identify graph properties

HIGH SCHOOL · FM.N.A.16

FM.N.A.16 "Use vertex and edge graphs to model mathematical situations involving networks. a. Identify properties of simple graphs, complete graphs, bipartite graphs, complete bipartite graphs, and trees."

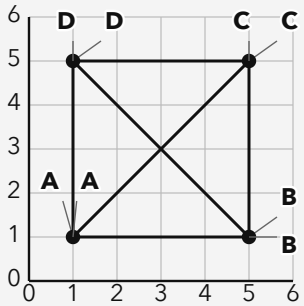
NAME _____ DATE _____

COUNTING CONNECTIONS

A simple graph has no loops or repeated edges. In a complete graph, every pair of vertices is joined, so n vertices have n(n - 1)/2 edges. A bipartite graph joins vertices only across two groups; a complete bipartite graph with groups of a and b has ab edges, while a tree with n vertices has n - 1 edges and no cycles.

WORKED EXAMPLE

How many edges are shown? Is this graph complete?

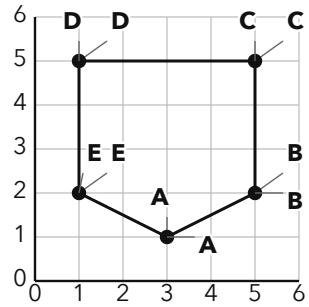


1. Count 4 vertices.
2. Each vertex is joined to the other 3, giving 4 × 3 = 12 edge ends.
3. Each edge has two ends, so 12/2 = 6 edges.
4. Every pair of vertices is joined.

Answer: 6 edges; yes, it is complete.

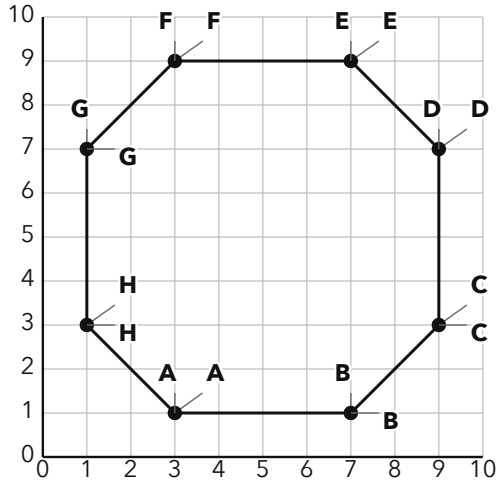
PRACTICE 6 problems

1 How many edges are shown? Is this graph bipartite?



ANSWER _____

2 How many edges are shown? Is this graph bipartite?



ANSWER _____

3 A complete graph has 7 vertices. How many edges does it have?

ANSWER _____

4 A tree has 9 vertices. How many edges does it have?

ANSWER _____

5 A complete bipartite graph has groups of 2 and 5 vertices. How many edges does it have?

ANSWER _____

6 A simple graph has 8 vertices and 28 edges. Must it be complete? Explain briefly.

ANSWER _____

APPLY IT Show your work

1 Ten students form a study group. Each pair plans one direct check-in. Model each student as a vertex and each planned check-in as an edge. How many edges are in the graph?

WORK SPACE

ANSWER _____

2 For a talent show, each of 2 singers rehearses once with each of 7 guitarists. No two singers rehearse together, and no two guitarists rehearse together. Model performers as vertices and rehearsal pairings as edges. How many edges are there, and is the graph complete bipartite?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A network with 11 vertices is a tree. How many edges must be added to make it a complete graph? Explain using the edge counts for both graphs.

WORK SPACE AND ANSWER

ANSWER KEY

Network Connections

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	How many edges are shown? Is this graph bipartite?	**5 edges; no. A cycle with an odd number of edges is not bipartite.**
2	How many edges are shown? Is this graph bipartite?	**8 edges; yes. Alternating vertices around the cycle form two groups with no edges within either group.**
3	A complete graph has 7 vertices. How many edges does it have?	**21 edges**
4	A tree has 9 vertices. How many edges does it have?	**8 edges**
5	A complete bipartite graph has groups of 2 and 5 vertices. How many edges does it have?	**10 edges**
6	A simple graph has 8 vertices and 28 edges. Must it be complete? Explain briefly.	**Yes. There are 8 × 7/2 = 28 possible pairs of vertices, so every pair must be joined.**

PART 2 · APPLY IT

1. **45 edges** Every pair is joined, so the graph is complete. Calculate 10 × 9/2 = 45.
2. **14 edges; yes, it is complete bipartite.** The singers and guitarists form two groups, with every cross-group pair joined. Calculate 2 × 7 = 14.

CHALLENGE

45 edges. The tree has 11 - 1 = 10 edges, and a complete graph on 11 vertices has 11 × 10/2 = 55 edges. Add 55 - 10 = 45 edges.

For bipartite graphs, accept any correct division of vertices into two groups with no edges within a group. For the final practice item, accept any explanation showing that 28 is the maximum number of edges on 8 vertices.