LESSON MAKER PRO

Count the Connections

Use vertex degrees to prove facts about networks.

HIGH SCHOOL · FM.N.A.17.b

FM.N.A.17.b "Give an argument for graph properties."

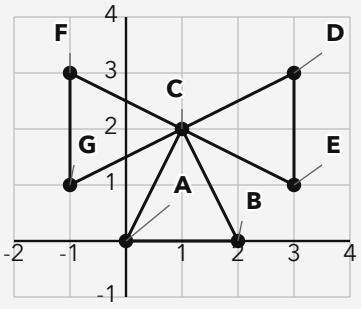
NAME _____ DATE _____

THE DEGREE RULE

A vertex's degree is the number of edges that meet there. Every edge adds 1 to the degree of each endpoint, so the sum of all degrees is twice the number of edges. In the diagrams, count labeled points as vertices and drawn polygon sides as edges, but do not count grid lines.

WORKED EXAMPLE

How many edges are in this network? Use the vertex degrees to explain.

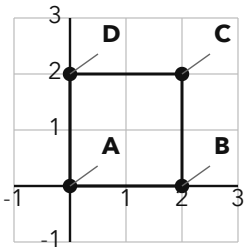


1. Vertex C has degree 6.
2. The other six vertices each have degree 2.
3. The degree sum is 6 + 6 × 2 = 18.
4. Each edge was counted twice, so 18 / 2 = 9.

Answer: 9 edges

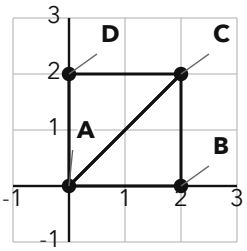
PRACTICE 6 problems

1 How many edges are in this network? Show how the degrees prove your answer.



ANSWER _____

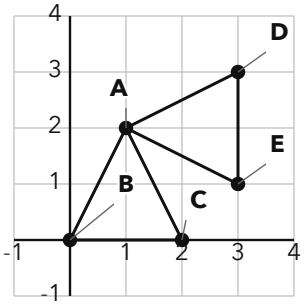
2 How many edges are in this network? Explain using the degrees. Count a side shared by two polygons only once.



ANSWER _____

_____

3 How many edges are in this network? Show your degree sum.



ANSWER _____

4 Could a network have four vertices with degrees 1, 1, 1, and 2? Explain using the degrees sum.

ANSWER _____

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5 A network has five vertices with degrees 4, 4, 2, 2, and 2. How many edges does it have? Show your calculation.

ANSWER _____

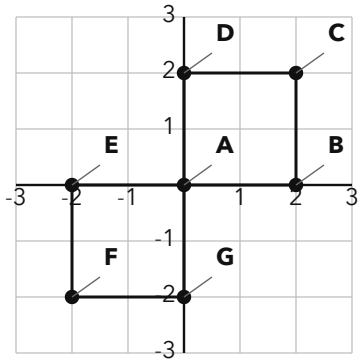
6 A student claims that a network can have exactly three vertices of odd degree. Is this possible? Explain.

ANSWER _____

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APPLY IT Show your work

1 At a school esports event, each edge shows one scheduled match between the teams at its endpoints. How many matches are scheduled? Prove your answer using the team degrees.



ANSWER _____

2 Five podcast club hosts have recorded episodes with 3, 3, 2, 2, and 2 other hosts, respectively. Each pair of hosts recorded one episode together. How many episodes did they record? Justify your answer.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A network has six vertices with degrees 4, 3, 3, 2, 2, and x. It has 8 edges. Find x, then explain why x could not be odd even if you did not know the number of edges.

WORK SPACE AND ANSWER

Count the Connections

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	How many edges are in this network? Show how the degrees prove your answer.	**(2 + 2 + 2 + 2) / 2 = 4 edges.**
2	How many edges are in this network? Explain using the degrees. Count a side shared by two polygons only once.	**The degrees are 3, 2, 3, and 2. Their sum is 10, so the network has 10 / 2 = 5 edges.**
3	How many edges are in this network? Show your degree sum.	**(4 + 2 + 2 + 2 + 2) / 2 = 6 edges.**
4	Could a network have four vertices with degrees 1, 1, 1, and 2? Explain using the degree sum.	**No. The degree sum would be 5, but every edge adds 2 to the degree sum, so the sum must be even.**
5	A network has five vertices with degrees 4, 4, 2, 2, and 2. How many edges does it have? Show your calculation.	**(4 + 4 + 2 + 2 + 2) / 2 = 7 edges.**
6	A student claims that a network can have exactly three vertices of odd degree. Is this possible? Explain.	**No. Three odd degrees would make the total degree sum odd, but every network's degree sum is twice its number of edges and must be even.**

PART 2 · APPLY IT

1. **8 matches** Team A has degree 4, and the other six teams each have degree 2. The degree sum is 4 + 6 × 2 = 16, so there are 16 / 2 = 8 matches.
2. **6 episodes** The degree sum is 3 + 3 + 2 + 2 + 2 = 12. Each episode includes two hosts, so 12 / 2 = 6 episodes.

CHALLENGE

x = 2. The known degrees sum to 14, and 8 edges require a degree sum of 16, so x = 2. Even without the edge count, the total degree sum must be even; since 14 is even, x cannot be odd.

Accept degree sums written in any order. For explanations, students should state that each edge contributes 2 to the degree sum.