LESSON MAKER PRO

Routes Without Repeats

Find and defend routes through networks.

HIGH SCHOOL · FM.N.A.17

FM.N.A.17 "Solve problems involving networks through investigation and application of existence and nonexistence of Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. a..."

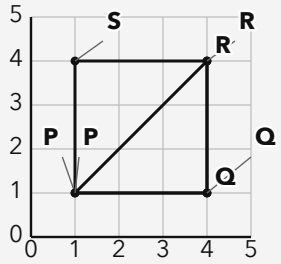
NAME _____ DATE _____

CHECK THE DEGREES

In a connected network, an Euler route uses every edge exactly once. If every vertex has even degree, the route can end where it starts. If exactly two vertices have odd degree, the route starts at one and ends at the other. With more than two odd vertices, no Euler route exists.

WORKED EXAMPLE

Can you use every edge in this network once? Give a route.



1. Count each drawn edge once.
2. P and R have odd degree. Q and S have even degree.
3. Start at P, end at R, and use each edge once.

Answer: Yes: P, Q, R, S, P, R.

PRACTICE 4 problems

1 A connected network has degrees 2, 2, 2, 2. Can an Euler route end where it starts?

ANSWER _____

2 A connected network has degrees 1, 1, 2, 2. How many possible starting vertices can an Euler route have?

ANSWER _____

3 A connected network has degrees 3, 3, 3, 3, 3, 3. Can you use every edge exactly once? Why?

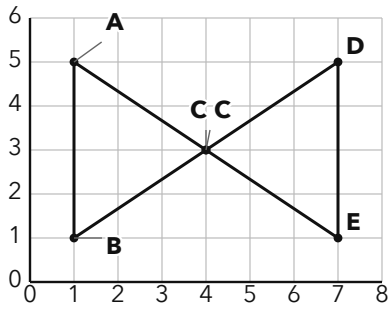
ANSWER _____

4 A connected network has degrees 4, 2, 2, 2, 2. Is an Euler circuit possible? Why?

ANSWER _____

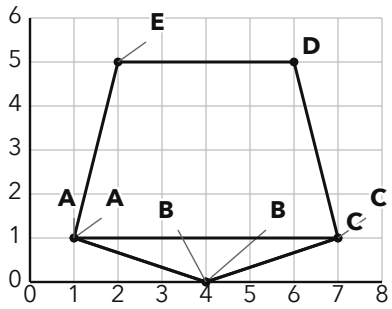
APPLY IT Show your work

1 A museum robot starts at C. It must inspect each marked corridor once and return to C. Give its route.



ANSWER _____

2 A street team starts at A and finishes at C. It must cover every marked street once. Give its route.



ANSWER _____

CHALLENGE One step up

Use the street network in Apply 2. The team must now start and finish at A while covering every street. What is the fewest street traversals? Give a route and explain why fewer cannot work.

WORK SPACE AND ANSWER

ANSWER KEY

Routes Without Repeats

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A connected network has degrees 2, 2, 2, 2. Can an Euler route end where it starts?	**Yes, it has an Euler circuit.**
2	A connected network has degrees 1, 1, 2, 2. How many possible starting vertices can an Euler route have?	**2**
3	A connected network has degrees 3, 3, 3, 3, 3, 3. Can you use every edge exactly once? Why?	**No. It has six odd-degree vertices.**
4	A connected network has degrees 4, 2, 2, 2, 2. Is an Euler circuit possible? Why?	**Yes. Every vertex has even degree.**

PART 2 · APPLY IT

1. **C, A, B, C, D, E, C.** Every vertex has even degree, so a circuit is possible. The route uses each of the six corridors once.
2. **A, B, C, D, E, A, C.** A and C are the only odd-degree vertices, so they can be the endpoints. The route covers all six streets once.

CHALLENGE

7 traversals: A, B, C, D, E, A, C, A. A and C have odd degree, so a closed route must repeat at least one street. Repeating A to C once reaches that minimum.

Accept any route that meets the stated endpoints and edge-use rules. Shared polygon sides in the figures represent one edge, not two.