LESSON MAKER PRO

Connect for Less

Find the lowest-cost links that reach every point.

HIGH SCHOOL · FM.N.A.18.b

FM.N.A.18.b "Show that every connected graph has a minimal spanning tree."

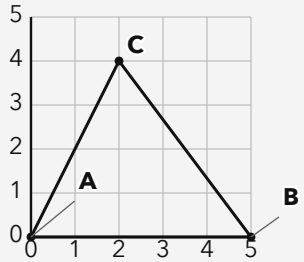
NAME _____ DATE _____

THE BIG IDEA

A spanning tree connects every point without a loop. In a finite, connected network, you can find a minimum spanning tree by adding the cheapest link that joins two separate groups. Stop when every point is connected.

WORKED EXAMPLE

Use the network. Link costs are AB 11, BC 13, and AC 17. Find a minimum spanning tree and its cost.



1. Choose AB, the cheapest link, for 11.
2. Add BC for 13 to connect the last point.
3. Stop. All three points are connected without a loop.

Answer: AB and BC; total cost 24.

PRACTICE 4 problems

1 Links (cost): AB(2), BC(5), AC(8). Find a minimum spanning tree and its cost.

ANSWER _____

2 Links (cost): AB(1), BC(4), CD(2), DA(6), AC(3). Find a minimum spanning tree and its cost.

ANSWER _____

3 Links (cost): AB(3), AC(3), BC(6), CD(2), BD(5). Find a minimum spanning tree and its cost.

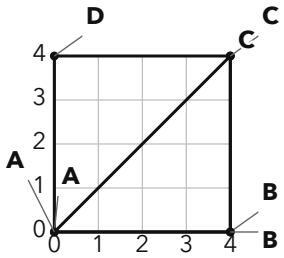
ANSWER _____

4 Links (cost): AB(4), AC(7), BC(2), BD(6), CD(3), CE(8), DE(5). Find a minimum spanning tree and its cost.

ANSWER _____

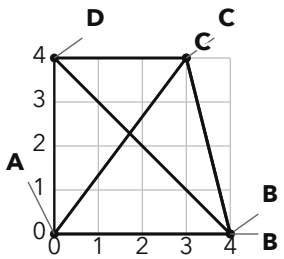
APPLY IT Show your work

1 A gaming club needs cables between four stations. Use the network. Cable costs in dollars: AB 6, BC 2, CD 7, DA 4, AC 5. Which cables cost the least while reaching every station?



ANSWER _____

2 A festival needs Wi-Fi links between four booths. Use the network. Link costs in dollars: AB 3, AC 4, BC 8, BD 6, CD 2. Which links cost the least while reaching every booth?



ANSWER _____

CHALLENGE One step up

A connected network has positive-cost links AB(2), BC(4), CD(5), DA(8), AC(3). Find its least-cost tree. Explain why a least-cost connected plan cannot contain a loop.

WORK SPACE AND ANSWER

Connect for Less

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Links (cost): AB(2), BC(5), AC(8). Find a minimum spanning tree and its cost.	**AB and BC; total cost 7.**
2	Links (cost): AB(1), BC(4), CD(2), DA(6), AC(3). Find a minimum spanning tree and its cost.	**AB, CD, and AC; total cost 6.**
3	Links (cost): AB(3), AC(3), BC(6), CD(2), BD(5). Find a minimum spanning tree and its cost.	**CD, AB, and AC; total cost 8.**
4	Links (cost): AB(4), AC(7), BC(2), BD(6), CD(3), CE(8), DE(5). Find a minimum spanning tree and its cost.	**BC, CD, AB, and DE; total cost 14.**

PART 2 · APPLY IT

1. **BC, DA, and AC; total cost $11.** BC and DA connect two pairs of stations. AC joins the pairs for $5, giving a total of $11.
2. **CD, AB, and AC; total cost $9.** Choose CD for $2 and AB for $3. AC joins the two pairs for $4, giving a total of $9.

CHALLENGE

AB, AC, and CD form a tree with cost 10. If a connected plan contains a loop, removing one link from that loop keeps every point connected and lowers the cost.

Accept links in any order. A correct loop explanation must say that removing a link from the loop preserves connectivity and reduces cost.