LESSON MAKER PRO

Connect for Less

Find the lowest-cost way to connect every point.

HIGH SCHOOL · FM.N.A.18.c

FM.N.A.18.c "Use Kruskal's Algorithm and Prim's Algorithm to determine the minimal spanning tree of a weighted graph."

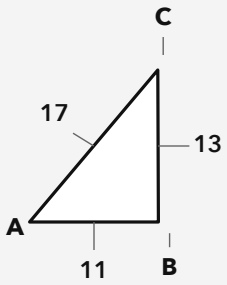
NAME _____ DATE _____

TWO WAYS TO BUILD

A minimum spanning tree connects every vertex without making a cycle. Kruskal's Algorithm checks edges from least to greatest weight, adding one unless it makes a cycle. Prim's Algorithm starts at one vertex and repeatedly adds the cheapest edge to a new vertex.

WORKED EXAMPLE

Use Prim's Algorithm starting at A. List the tree edges and total weight.



1. Start at A. Add AB, weight 11.
2. From A or B, add BC, weight 13.
3. All vertices are connected. Add 11 + 13.

Answer: AB, BC; total weight 24

PRACTICE 4 problems

1 Edges (weight): AB 1, AC 3, BC 5, CD 6, AD 9. Use Kruskal's Algorithm. Give the tree edges and total weight.

ANSWER _____

2 Edges (weight): AB 3, AC 6, BC 1, BD 8, CD 5. Use Prim's Algorithm starting at A. Give the tree edges and total weight.

ANSWER _____

3 Edges (weight): AB 6, AC 8, BC 3, BD 5, CD 9, AD 10. Use Kruskal's Algorithm. Give the tree edges and total weight.

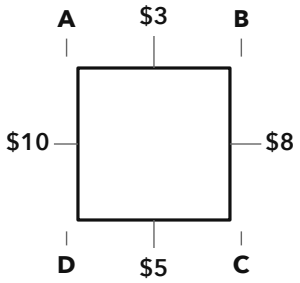
ANSWER _____

4 Edges (weight): AB 5, AC 9, AD 8, BC 1, BD 6, CD 3. Use Prim's Algorithm starting at A. Give the tree edges and total weight.

ANSWER _____

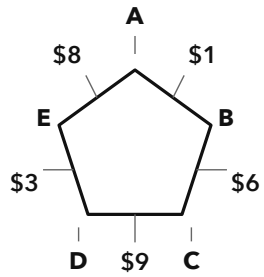
APPLY IT Show your work

1 A gaming club needs cables connecting stations A, B, C, and D. The figure shows each possible cable's cost in dollars. Use Kruskal's Algorithm to find the least-cost set of cables.



ANSWER _____

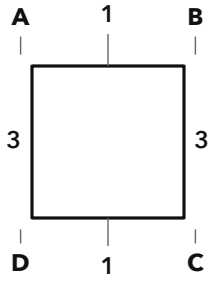
2 A school festival needs lights connecting booths A through E. The figure shows the cost in dollars for each possible connection. Use Prim's Algorithm starting at A to find the least-cost set.



ANSWER _____

CHALLENGE One step up

Use Prim's Algorithm starting at A and Kruskal's Algorithm to find two different minimum spanning trees. List each tree and explain why both are minimum.



WORK SPACE AND ANSWER

Connect for Less

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Edges (weight): AB 1, AC 3, BC 5, CD 6, AD 9. Use Kruskal's Algorithm. Give the tree edges and total weight.	**AB, AC, CD; total weight 10**
2	Edges (weight): AB 3, AC 6, BC 1, BD 8, CD 5. Use Prim's Algorithm starting at A. Give the tree edges and total weight.	**AB, BC, CD; total weight 9**
3	Edges (weight): AB 6, AC 8, BC 3, BD 5, CD 9, AD 10. Use Kruskal's Algorithm. Give the tree edges and total weight.	**BC, BD, AB; total weight 14**
4	Edges (weight): AB 5, AC 9, AD 8, BC 1, BD 6, CD 3. Use Prim's Algorithm starting at A. Give the tree edges and total weight.	**AB, BC, CD; total weight 9**

PART 2 · APPLY IT

1. **AB, BC, CD; $16** Choose the three cheapest edges. They connect every station for $3 + $8 + $5 = $16.
2. **AB, BC, AE, DE; $18** From A, add AB, then BC, then AE, then ED. The cost is $1 + $6 + $8 + $3 = $18.

CHALLENGE

One Prim tree is AB, BC, CD. One Kruskal tree is AB, CD, DA. Each has total weight 5. Both use the two weight-1 edges and one weight-3 edge to connect all four vertices, so neither can cost less.

Accept tree edges in any order. For the challenge, accept any two distinct minimum spanning trees with a valid explanation.