LESSON MAKER PRO

Connect for Less

Find the lowest-cost network.

HIGH SCHOOL · FM.N.A.18

FM.N.A.18 "Apply algorithms relating to minimum weight spanning trees, networks, flows, and Steiner trees. a. Use shortest path techniques to find optimal shipping routes. b. Show that every connected graph has a minimal spanning tree. c..."

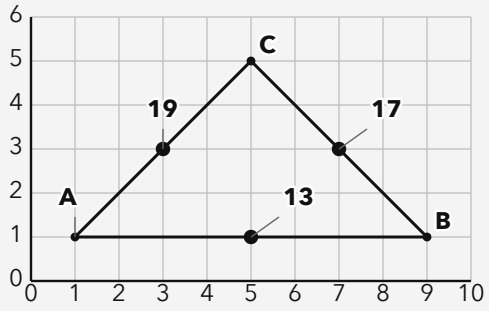
NAME _____ DATE _____

KRUSKAL'S RULE

To connect every site at the lowest total weight, consider edges from least to greatest weight. Add an edge only if it does not make a cycle. Stop when you have one fewer edge than sites.

WORKED EXAMPLE

Lettered points are sites. Numbers by the edges are weights. Use Kruskal's algorithm to connect all three sites.



1. Choose AB, the edge with weight 13.
2. Choose BC, the edge with weight 17. It makes no cycle.
3. Stop: three sites need two edges.

Answer: AB and BC; total weight 30.

PRACTICE 4 problems

1 Use Kruskal's algorithm. Edges: AB=1, AC=8, BC=3, BD=7, CD=4. Give the chosen edges and total weight.

ANSWER _____

2 Use Kruskal's algorithm. Edges: AB=4, AC=8, AD=6, BC=2, BD=7, CD=3. Give the chosen edges and total weight.

ANSWER _____

3 Use Kruskal's algorithm. Edges: AB=3, AC=9, AD=10, BC=5, BD=8, CD=2. Give the chosen edges and total weight.

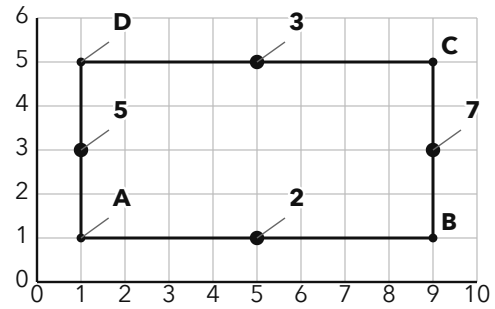
ANSWER _____

4 Use Kruskal's algorithm. Edges: AB=2, AC=4, AD=7, BC=3, BD=6, CD=5. Give the chosen edges and total weight.

ANSWER _____

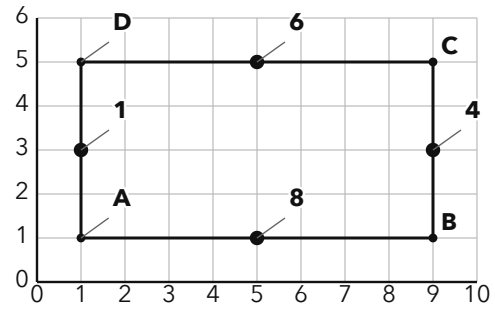
APPLY IT Show your work

1 A school is laying cable between four campus buildings. The numbers by the lines show costs in thousands of dollars. Which connections cost the least while linking every building?



ANSWER _____

2 Friends are planning paths between four areas of a community garden. The numbers by the lines show building costs in hundreds of dollars. Which paths link all areas for the least cost?



ANSWER _____

CHALLENGE One step up

Explain why every finite, connected network has a minimum-weight spanning tree.

WORK SPACE AND ANSWER

Connect for Less

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use Kruskal's algorithm. Edges: AB=1, AC=8, BC=3, BD=7, CD=4. Give the chosen edges and total weight.	**AB, BC, CD; total weight 8.**
2	Use Kruskal's algorithm. Edges: AB=4, AC=8, AD=6, BC=2, BD=7, CD=3. Give the chosen edges and total weight.	**BC, CD, AB; total weight 9.**
3	Use Kruskal's algorithm. Edges: AB=3, AC=9, AD=10, BC=5, BD=8, CD=2. Give the chosen edges and total weight.	**CD, AB, BC; total weight 10.**
4	Use Kruskal's algorithm. Edges: AB=2, AC=4, AD=7, BC=3, BD=6, CD=5. Give the chosen edges and total weight.	**AB, BC, CD; total weight 10.**

PART 2 · APPLY IT

1. **AB, CD, and DA; $10,000.** Choose weights 2, 3, and 5. These three connections link all four buildings without a cycle.
2. **DA, BC, and CD; $1,100.** Choose weights 1, 4, and 6. They link all four areas without a cycle.

CHALLENGE

Remove an edge from any cycle, repeating until no cycles remain. Removing a cycle edge keeps the network connected, so the result is a spanning tree. A finite network has finitely many spanning trees, and one of them has the smallest total weight.

Accept chosen edges in any order. In the challenge, accept any valid argument that establishes both the existence of a spanning tree and a least-weight choice among finitely many spanning trees.