LESSON MAKER PRO

Count the Routes

Use matrix powers to count walks.

HIGH SCHOOL · FM.N.A.21

FM.N.A.21 "Use the adjacency matrix of a graph to determine the number of walks of length n in a graph."

NAME _____ DATE _____

THE BIG IDEA

An adjacency matrix shows which vertices are connected. In M^n, the entry in row A and column B counts the walks of length n from A to B.

WORKED EXAMPLE

Using the matrix, how many 3-step walks go from P to Q?

M

	P	Q	R
P	0	1	0
Q	1	0	1
R	0	1	0

1. The P row of M^2 is [1, 0, 1].
2. For the P,Q entry of M^3, multiply that row by column Q of M.
3. $1 \times 1 + 0 \times 0 + 1 \times 1 = 2$.

Answer: 2 walks

PRACTICE 4 problems

1 M = [[0, 1], [1, 0]], with rows and columns A, B. How many 2-step walks go from A to A?

ANSWER _____

2 M = [[0, 1, 1], [1, 0, 0], [1, 0, 0]], with rows and columns A, B, C. How many 2-step walks go from B to C?

ANSWER _____

3 M = [[0, 1, 1], [0, 0, 1], [0, 0, 0]], with rows and columns A, B, C. How many 2-step walks go from A to C?

ANSWER _____

4 M = [[0, 1, 1], [1, 0, 1], [1, 1, 0]], with rows and columns A, B, C. How many 2-step walks go from A to A?

ANSWER _____

APPLY IT Show your work

1 A music app uses the matrix to show which playlists link to others. Rows are starting playlists. How many ways can you follow two links from K back to K?

PLAYLIST LINKS

	K	L	M
K	0	1	1
L	1	0	1
M	1	0	0

ANSWER _____

2 At a school event, the matrix shows which stations have direct paths between them. Rows are starting stations. How many 2-path routes go from A to D?

STATION PATHS

	A	B	C	D
A	0	1	1	0
B	0	0	0	1
C	0	0	0	1
D	1	0	0	0

ANSWER _____

CHALLENGE One step up

Use M from problem 4. How many 4-step walks go from A back to A? Show a matrix calculation.

WORK SPACE AND ANSWER

Count the Routes

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	M = [[0, 1], [1, 0]], with rows and columns A, B. How many 2-step walks go from A to A?	**1**
2	M = [[0, 1, 1], [1, 0, 0], [1, 0, 0]], with rows and columns A, B, C. How many 2-step walks go from B to C?	**1**
3	M = [[0, 1, 1], [0, 0, 1], [0, 0, 0]], with rows and columns A, B, C. How many 2-step walks go from A to C?	**1**
4	M = [[0, 1, 1], [1, 0, 1], [1, 1, 0]], with rows and columns A, B, C. How many 2-step walks go from A to A?	**2**

PART 2 · APPLY IT

1. **2 ways** The K,K entry of M^2 is $0 \times 0 + 1 \times 1 + 1 \times 1 = 2$.
2. **2 routes** The A,D entry of M^2 is $0 \times 0 + 1 \times 1 + 1 \times 1 + 0 \times 0 = 2$.

CHALLENGE

6 walks. Row A of M^2 is [2, 1, 1], so the A,A entry of M^4 is $2 \times 2 + 1 \times 1 + 1 \times 1 = 6$.

For the challenge, accept any correct matrix multiplication that gives 6.