LESSON MAKER PRO

Patterns That Grow

Model change with recursive and explicit formulas.

HIGH SCHOOL · FM.R.A.11

FM.R.A.11 "Find patterns in application problems involving series and sequences, and develop recursive and explicit formulas as models to understand and describe sequential change."

NAME _____ DATE _____

TWO WAYS TO MODEL

A constant difference d gives an arithmetic sequence: $a_n=a_{n-1}+d$ and $a_n=a_1+(n-1)d$. A constant ratio r gives a geometric sequence: $a_n=r(a_{n-1})$ and $a_n=a_1r^{n-1}$. For either recursive rule, state a_1 and use the rule for $n\geq2$.

WORKED EXAMPLE

Use the table to write recursive and explicit formulas. Then find a_4.

n	a_n
1	16
2	22
3	28

1. 22-16=6 and 28-22=6, so d=6.
2. Recursive: $a_1=16$; $a_n=a_{n-1}+6$ for $n\geq2$.
3. Explicit: $a_n=16+6(n-1)$.
4. $a_4=16+6(4-1)=34$.

Answer: $a_1=16$; $a_n=a_{n-1}+6$ for $n\geq2$; $a_n=16+6(n-1)$; $a_4=34$.

PRACTICE 4 problems

1 For 3, 8, 13, ... write recursive and explicit formulas.

ANSWER _____

2 For 81, 27, 9, ... write recursive and explicit formulas.

ANSWER _____

3 Given $a_1=2$ and $a_n=a_{n-1}+4$, write an explicit formula and find a_9.

ANSWER _____

4 Given $a_1=5$ and $a_n=2a_{n-1}$, write an explicit formula and find a_4.

ANSWER _____

APPLY IT Show your work

1 In week 1, your running playlist lasts 32 minutes. You add 9 minutes of music each week. Write recursive and explicit formulas for its length in week n. How long is it in week 8?

WORK SPACE

ANSWER _____

2 On day 1, a video gets 2 new shares. Each day, it gets three times as many new shares as the day before. Write recursive and explicit formulas for new shares on day n. How many new shares occur on day 5?

WORK SPACE

ANSWER _____

CHALLENGE One step up

An arithmetic sequence has $a_3=23$ and $a_8=43$. Find recursive and explicit formulas and a_{10}. Explain how you found a_1.

WORK SPACE AND ANSWER

ANSWER KEY # Patterns That Grow

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For 3, 8, 13, ... write recursive and explicit formulas.	**$a_1=3$; $a_n=a_{n-1}+5$ for $n\geq 2$; $a_n=3+5(n-1)$.**
2	For 81, 27, 9, ... write recursive and explicit formulas.	**$a_1=81$; $a_n=(1/3)a_{n-1}$ for $n\geq 2$; $a_n=81(1/3)^{n-1}$.**
3	Given $a_1=2$ and $a_n=a_{n-1}+4$, write an explicit formula and find a_9.	**$a_n=2+4(n-1)$; $a_9=34$.**
4	Given $a_1=5$ and $a_n=2a_{n-1}$, write an explicit formula and find a_4.	**$a_n=5(2)^{n-1}$; $a_4=40$.**

PART 2 · APPLY IT

1. **$a_1=32$; $a_n=a_{n-1}+9$ for $n\geq 2$; $a_n=32+9(n-1)$; 95 minutes.** The length increases by 9 minutes each week. In week 8, it is 32+9(8-1)=95 minutes.
2. **$a_1=2$; $a_n=3a_{n-1}$ for $n\geq 2$; $a_n=2(3)^{n-1}$; 162 new shares.** New shares form a geometric sequence with ratio 3. On day 5, there are $2(3)^4=162$ new shares.

CHALLENGE

The difference is (43-23)/(8-3)=4. Since a_3 is two steps after a_1, $a_1=23-2(4)=15$. Recursive: $a_1=15$; $a_n=a_{n-1}+4$ for $n\geq 2$. Explicit: $a_n=15+4(n-1)$. Thus $a_{10}=51$.

Accept equivalent explicit formulas, such as 5n-2 for practice problem 1. Recursive formulas must include the first term.