LESSON MAKER PRO

Proof by Steps

Use induction to prove a claim for every positive integer.

HIGH SCHOOL · FM.R.A.14

FM.R.A.14 "Use mathematical induction to prove statements involving the positive integers."

NAME _____ DATE _____

THE INDUCTION CHAIN

First, check that the claim is true when n=1. Then assume it is true when n=k, and use that assumption to prove it when n=k+1. Together, these steps prove the claim for every positive integer.

WORKED EXAMPLE

Prove that $4+8+\ldots+4n=2n(n+1)$ for $n>=1$.

1. Base case, n=1: $4=2(1)(2)$.
2. Assume $4+8+\ldots+4k=2k(k+1)$.
3. Add the next term: $2k(k+1)+4(k+1)=2(k+1)(k+2)$.

Answer: The claim holds for n=1 and passes from k to k+1, so it holds for every n>=1.

PRACTICE 4 problems

1 Prove that $5+10+\ldots+5n=5n(n+1)/2$ for $n>=1$.

ANSWER _____

2 Prove that $1+3+\ldots+(2n-1)=n^2$ for $n>=1$.

ANSWER _____

3 Prove that $3+9+\ldots+3^n=(3^{n+1}-3)/2$ for $n>=1$.

ANSWER _____

4 Prove that 8^n-1 is divisible by 7 for $n>=1$.

ANSWER _____

APPLY IT Show your work

1 A theater has 9 seats in its first row and 2 more seats in each row after it. Prove that the first n rows have n(n+8) seats for n>=1.

WORK SPACE

ANSWER _____

2 A video gets 5 new views on day 1. Each day it gets twice as many new views as the day before. Prove that its total after n days is $5(2^n-1)$ for n>=1.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Prove that $n^2 <= 2^n$ for every integer n>=4.

WORK SPACE AND ANSWER

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Prove that $5+10+...+5n=5n(n+1)/2$ for $n>=1$.	**At n=1, $5=5(1)(2)/2$. If the claim holds for k, adding $5(k+1)$ gives $5k(k+1)/2+5(k+1)=5(k+1)(k+2)/2$, so induction proves the claim.**
2	Prove that $1+3+...+(2n-1)=n^2$ for $n>=1$.	**At n=1, $1=1^2$. If the claim holds for k, adding $2k+1$ gives $k^2+2k+1=(k+1)^2$, so induction proves the claim.**
3	Prove that $3+9+...+3^n=(3^{n+1}-3)/2$ for $n>=1$.	**At n=1, $3=(3^2-3)/2$. If the claim holds for k, adding 3^{k+1} gives $(3^{k+1}-3)/2+3^{k+1}=(3^{k+2}-3)/2$, so induction proves the claim.**
4	Prove that 8^n-1 is divisible by 7 for $n>=1$.	**At n=1, $8-1=7$. If 8^k-1 is divisible by 7, then $8^{k+1}-1=8(8^k-1)+7$ is also divisible by 7, so induction proves the claim.**

PART 2 · APPLY IT

1. **At n=1, $9=1(1+8)$. If the first k rows have $k(k+8)$ seats, the next row has $2k+9$ seats. Their new total is $k(k+8)+2k+9=(k+1)(k+9)$, as required.** Check the first row, then add the seats in row k+1 to the assumed total for k rows.
2. **At n=1, $5=5(2^1-1)$. If the total after k days is $5(2^k-1)$, day k+1 adds $5\cdot2^k$ views. The new total is $5(2^k-1)+5\cdot2^k=5(2^{k+1}-1)$.** Check day 1, then add day k+1's views to the assumed total after k days.

CHALLENGE

At n=4, $16=2^4$. Assume $k^2 <= 2^k$ for $k>=4$. Since $k^2 >= 2k+1$ for $k>=4$, $(k+1)^2 <= 2k^2 <= 2^{k+1}$. Induction proves the claim.

Accept equivalent induction proofs that check the correct base case and explicitly use the assumption for k to establish the claim for k+1.