LESSON MAKER PRO

# Circle Equation Quest

*Use right triangles to find circles on a coordinate plane.*

**HIGH SCHOOL · GDA.AF.C.6**

**GDA.AF.C.6** "Derive the equation of a circle of given center and radius using the Pythagorean Theorem. a. Given the endpoints of the diameter of a circle..."

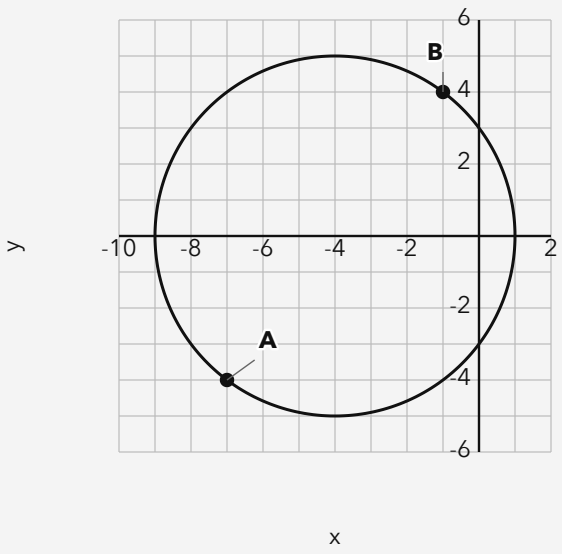
NAME _____ DATE _____

## FROM DIAMETER TO EQUATION

A circle with center (h, k) and radius r has the equation $(x - h)^2 + (y - k)^2 = r^2$. If you know the ends of a diameter, use their midpoint to find the center. The horizontal and vertical changes from the center to either endpoint form a right triangle, so the Pythagorean Theorem gives $r^2$.

### WORKED EXAMPLE

Points A and B are the ends of a diameter. Find the equation of the circle shown.

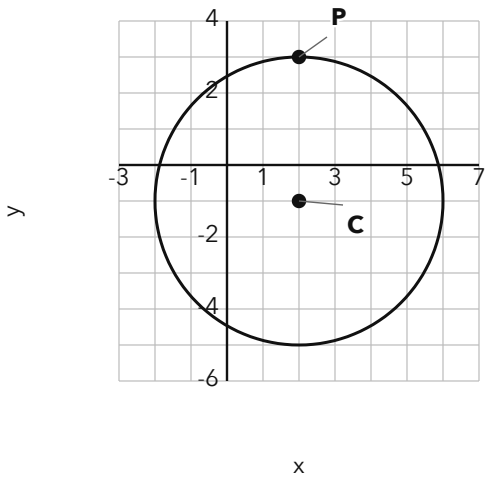



1. Find the midpoint: ((-7 + -1)/2, (-4 + 4)/2) = (-4, 0).
2. From the center to B, the horizontal change is 3 and the vertical change is 4.
3. Use the Pythagorean Theorem: $r^2 = 3^2 + 4^2 = 25$.

**Answer: $(x + 4)^2 + y^2 = 25$**

## PRACTICE 6 problems

**1** The circle is centered at C and passes through P. Write its equation.

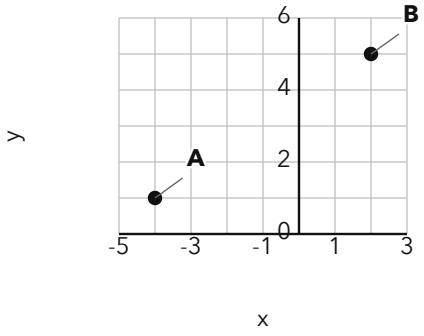



**ANSWER** _____

**2** Write the equation of a circle with center (-3, 2) and radius 6.

**ANSWER** _____

**3** Points A and B on the graph are the ends of a diameter. Write the equation of the circle.




**ANSWER** _____

**4** A circle has diameter endpoints (-1, -5) and (3, 3). Find its center and write its equation.

**ANSWER** _____

**5** Find the distance between (-6, -1) and (2, 5) using the Pythagorean Theorem.

**ANSWER** _____

**6** A circle has center (4, -2) and passes through (1, 3). Write its equation.

**ANSWER** _____

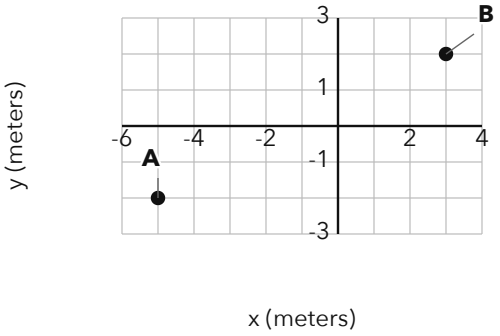
## APPLY IT Show your work

**1** A phone game awards a badge at locations exactly 6 meters from a checkpoint at (-2, 1) on a coordinate map. Each unit represents 1 meter. Write the equation for the badge boundary.

WORK SPACE

**ANSWER** _____

**2** An outdoor concert plans a circular light display. Markers A and B on the map are opposite ends of its diameter. Each unit represents 1 meter. Use the graph to write the equation of the display's edge.




**ANSWER** _____

## CHALLENGE One step up

Points (a, b) and (c, d) form opposite corners of a right triangle whose legs are horizontal and vertical. Use the Pythagorean Theorem to derive a formula for the distance between the points. Then use that formula to derive the equation of a circle centered at (h, k) with radius r. Show both equations.

WORK SPACE AND ANSWER

**ANSWER KEY**

# Circle Equation Quest

**HIGH SCHOOL · GDA.AF.C.6**

## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | The circle is centered at C and passes through P. Write its equation. | **$(x - 2)^2 + (y + 1)^2 = 16$** |
| 2 | Write the equation of a circle with center (-3, 2) and radius 6. | **$(x + 3)^2 + (y - 2)^2 = 36$** |
| 3 | Points A and B on the graph are the ends of a diameter. Write the equation of the circle. | **Center: (-1, 3). Equation: $(x + 1)^2 + (y - 3)^2 = 13$** |
| 4 | A circle has diameter endpoints (-1, -5) and (3, 3). Find its center and write its equation. | **Center: (1, -1). Equation: $(x - 1)^2 + (y + 1)^2 = 20$** |
| 5 | Find the distance between (-6, -1) and (2, 5) using the Pythagorean Theorem. | **10** |
| 6 | A circle has center (4, -2) and passes through (1, 3). Write its equation. | **$(x - 4)^2 + (y + 2)^2 = 34$** |

## PART 2 · APPLY IT

1. **$(x + 2)^2 + (y - 1)^2 = 36$** The checkpoint is the center and the radius is 6 meters. Substitute these values into the circle equation.
2. **$(x + 1)^2 + y^2 = 20$** The midpoint is (-1, 0). From the midpoint to B, the changes are 4 and 2, so $r^2 = 4^2 + 2^2 = 20$.

## CHALLENGE

**The leg lengths are $|c - a|$ and $|d - b|$, so distance = $\sqrt{((c - a)^2 + (d - b)^2)}$. A point (x, y) is on the circle when its distance from (h, k) is r, so $r = \sqrt{((x - h)^2 + (y - k)^2)}$. Squaring gives $(x - h)^2 + (y - k)^2 = r^2$.**

Accept equivalent equations written in expanded form. For diameter problems, the radius may be left as a square root, but the equation must use the correct value of $r^2$.