LESSON MAKER PRO

Fit Check

Use residuals to compare and improve linear models.

HIGH SCHOOL · GDA.DSP.D.12.a

GDA.DSP.D.12.a "Find a linear function for a scatter plot that suggests a linear association and informally assess its fit by plotting and analyzing residuals, including the squares of the residuals, in order to improve its fit."

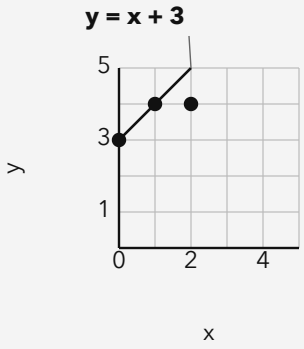
NAME _____ DATE _____

CHECK THE ERRORS

A residual is actual y minus predicted y. Plot each residual against its x-value: a pattern can show that a line misses the data in a consistent way. To compare lines, square each residual and add the squares; the smaller sum indicates a better fit to those points.

WORKED EXAMPLE

Compare y = x + 3 with y = x + 4 for the plotted points.



1. For y = x + 3, the residuals at x = 0, 1, 2 are 0, 0, -1. Plot them at (0, 0), (1, 0), (2, -1).
2. Its sum of squared residuals is 0 + 0 + 1 = 1.
3. For y = x + 4, the residuals are -1, -1, -2. Their squares sum to 1 + 1 + 4 = 6.

Answer: y = x + 3 fits these points better because 1 < 6.

PRACTICE 4 problems

1 For the point (2, 7), use y = 2x + 2. Find the residual and its square.

ANSWER _____

2 For (1, 5), (2, 6), (3, 9), find the sum of squared residuals for y = 2x + 2.

ANSWER _____

3 For (1, 4), (2, 5), (3, 6), compare y = x + 3 and y = x + 2. Give each sum of squared residuals and choose the better fit.

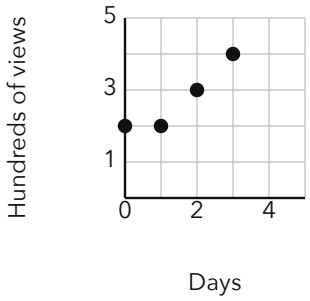
ANSWER _____

4 For (1, 3), (2, 5), (3, 7), find the sum of squared residuals for y = x + 2. Then give a line with a sum of 0.

ANSWER _____

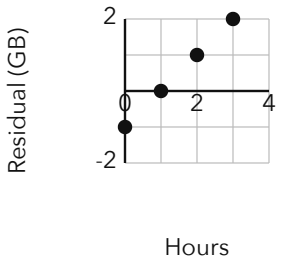
APPLY IT Show your work

1 The plot shows days since a video was posted and its views, in hundreds. Which model fits better, y = x + 1 or y = x + 2? Compare their sums of squared residuals.



ANSWER _____

2 The plot shows residuals, in GB, for y = x + 2, a model of total phone data used after x hours of streaming. Find the sum of squared residuals. Use the pattern to give a line that fits all four observations.



ANSWER _____

CHALLENGE One step up

For (0, 2), (1, 5), (2, 6), (3, 9), compare y = 2x + 2 with y = 2x + 5/2 using sums of squared residuals. Explain why one fits better even though it misses every point.

WORK SPACE AND ANSWER

Fit Check

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For the point (2, 7), use y = 2x + 2. Find the residual and its square.	**Residual: 1; square: 1.**
2	For (1, 5), (2, 6), (3, 9), find the sum of squared residuals for y = 2x + 2.	**2**
3	For (1, 4), (2, 5), (3, 6), compare y = x + 3 and y = x + 2. Give each sum of squared residuals and choose the better fit.	**y = x + 3: 0; y = x + 2: 3. Better fit: y = x + 3.**
4	For (1, 3), (2, 5), (3, 7), find the sum of squared residuals for y = x + 2. Then give a line with a sum of 0.	**Sum: 5; line: y = 2x + 1.**

PART 2 · APPLY IT

1. **y = x + 1 fits better; its sum is 1, versus 3 for y = x + 2.** The residuals are 1, 0, 0, 0 for y = x + 1 and 0, -1, -1, -1 for y = x + 2.
2. **Sum: 6; improved line: y = 2x + 1.** The squares add to 1 + 0 + 1 + 4 = 6. Adding each residual to its prediction gives observations (0, 1), (1, 3), (2, 5), (3, 7), all on y = 2x + 1.

CHALLENGE

The sums are 2 for y = 2x + 2 and 1 for y = 2x + 5/2. The second line fits better because its smaller errors have a lower sum of squares, even though none of its residuals is 0.

Residuals use actual minus predicted. Accept equivalent line notation and explanations that correctly compare the sums of squared residuals.