LESSON MAKER PRO

Lines That Fit

Use residuals to judge a linear model.

HIGH SCHOOL · GDA.DSP.E

GDA.DSP.E "Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns..."

NAME _____ **DATE** _____

THE BIG IDEA

For three points with equally spaced x-values, the least-squares slope is (last y - first y)/(last x - first x). The line passes through (mean x, mean y). A residual is actual y minus predicted y, and SSE is the sum of the squared residuals.

WORKED EXAMPLE

Find the least-squares line and SSE for the data.

x	y
8	17
10	18
12	25

1. Slope = (25 - 17)/(12 - 8) = 2.
2. The means are x = 10 and y = 20, so the intercept is 20 - 2(10) = 0.
3. Predictions are 16, 20, 24. Residuals are 1, -2, 1.
4. SSE = $1^2 + (-2)^2 + 1^2 = 6$.

Answer: Line: y = 2x. SSE: 6.

PRACTICE 4 problems

1 Find the least-squares line for (1, 4), (2, 9), (3, 11).

ANSWER _____

2 For $y = 3x + 1$ and data (0, 2), (1, 3), (2, 8), find the residuals and SSE.

ANSWER _____

_____

3 For (0, 1), (1, 4), (2, 5), find each SSE. Which fits better: $y = x + 1$ or $y = 2x + 1$?

ANSWER _____

_____

4 For (0, 5), (1, 6), (2, 10), find the middle point's residual from the least-squares line.

ANSWER _____

APPLY IT Show your work

1 The table shows study hours and quiz scores. Find the least-squares line and predict the score for 4 hours. Do these data prove that studying caused the scores? Give one reason.

STUDY AND QUIZ SCORES

Hours, x	Score, y
1	61
2	71
3	75

ANSWER _____

2 The table pairs daily screen time with sleep time. Find the least-squares line and its SSE.

SCREEN TIME AND SLEEP

Screen hours, x	Sleep hours, y
1	9
2	6
3	6

ANSWER _____

CHALLENGE One step up

For (0, 2), (1, 6), (2, 7), find the least-squares line. Compare its SSE with the SSE for $y = 2x + 3$. Which line fits better?

WORK SPACE AND ANSWER

Lines That Fit

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the least-squares line for (1, 4), (2, 9), (3, 11).	**y = (7/2)x + 1**
2	For y = 3x + 1 and data (0, 2), (1, 3), (2, 8), find the residuals and SSE.	**Residuals: 1, -1, 1. SSE: 3.**
3	For (0, 1), (1, 4), (2, 5), find each SSE. Which fits better: y = x + 1 or y = 2x + 1?	**SSEs: 8 and 1, respectively. y = 2x + 1 fits better.**
4	For (0, 5), (1, 6), (2, 10), find the middle point's residual from the least-squares line.	**-1**

PART 2 · APPLY IT

1. **y = 7x + 55; predicted score: 83. No. Other factors, such as prior knowledge, could affect scores.** The slope is (75 - 61)/(3 - 1) = 7. The means are 2 and 69, giving an intercept of 55.
2. **y = (-3/2)x + 10; SSE = 3/2.** The slope is (6 - 9)/(3 - 1) = -3/2, and the means are 2 and 7. Residuals are 1/2, -1, 1/2, so SSE = 1/4 + 1 + 1/4 = 3/2.

CHALLENGE

The least-squares line is y = (5/2)x + 5/2, with SSE = 3/2. The line y = 2x + 3 has SSE = 2, so the least-squares line fits better.

Accept equivalent forms of the equations and SSE values. Residuals must use actual y minus predicted y.