LESSON MAKER PRO

Solid Reasoning

Use simpler solids to explain area and volume.

HIGH SCHOOL · GDA.GM.A.17.a

GDA.GM.A.17.a "Give an informal argument for the formulas for the surface area and volume of a sphere, cylinder, pyramid, and cone using dissection arguments, Cavalieri's Principle, and informal limit arguments."

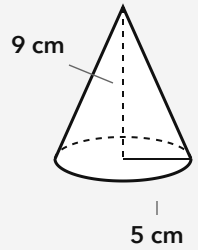
NAME _____ DATE _____

THE BIG IDEA

Unrolling a curved surface can reveal a simpler shape whose area you can find. A cone or pyramid has one-third the volume of a matching cylinder or prism. Comparing thin slices can also help explain a sphere's volume.

WORKED EXAMPLE

Find the volume of the cone shown by comparing it with a cylinder that has the same radius and height.

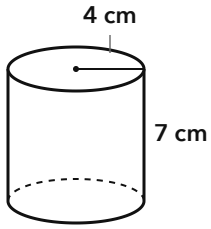


1. Matching cylinder: $V = \pi r^2 h$
2. $V = \pi(5)^2(9) = 225\pi$ cm^3
3. Cone: $V = 1/3(225\pi) = 75\pi$ cm^3

Answer: 75π cm^3

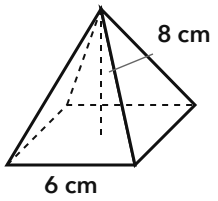
PRACTICE 6 problems

1 Imagine unrolling the side of the cylinder shown into a rectangle. Find the total area of its side and two circular ends.



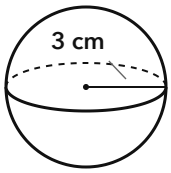
ANSWER _____

2 The figure shows a square pyramid. Compare it with a prism that has the same base and height to find its volume.



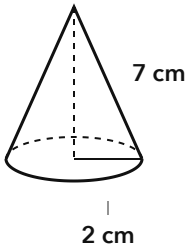
ANSWER _____

3 Find the volume of the sphere shown.



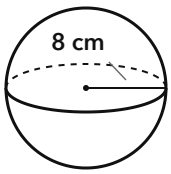
ANSWER _____

4 Imagine unrolling the curved side of the cone shown into a sector of a circle. Find the area of that curved side only.



ANSWER _____

5 Find the surface area of the sphere shown.



ANSWER _____

6 A cube with volume 216 cm^3 is cut into three congruent pyramids. They share a tip at one corner of the cube, and each has an opposite face as its base. Find one pyramid's volume and explain your division.

ANSWER _____

_____

APPLY IT Show your work

1 A closed cylindrical drink can for a school esports event has a radius of 3 cm and a height of 10 cm. How much metal covers its side and both ends, ignoring seams? Use an unrolled side to organize your calculation.

WORK SPACE

ANSWER _____

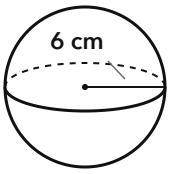
2 A cone-shaped prop for a video has a base radius of 4 cm and a vertical height of 12 cm. How much space is inside it? Compare its volume with a cylinder that has the same base and height.

WORK SPACE

ANSWER _____

CHALLENGE One step up

The sphere shown and a cylinder with two cones removed have the same height and the same area at every horizontal slice. The cylinder's radius is the sphere's radius, and its height is the sphere's diameter. Each removed cone has a base radius and height equal to the sphere's radius. Explain why the remaining cylinder and sphere have equal volume, then find the sphere's volume.



WORK SPACE AND ANSWER

Solid Reasoning

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Imagine unrolling the side of the cylinder shown into a rectangle. Find the total area of its side and two circular ends.	**88π cm^2**
2	The figure shows a square pyramid. Compare it with a prism that has the same base and height to find its volume.	**96 cm^3**
3	Find the volume of the sphere shown.	**36π cm^3**
4	Imagine unrolling the curved side of the cone shown into a sector of a circle. Find the area of that curved side only.	**14π cm^2**
5	Find the surface area of the sphere shown.	**256π cm^2**
6	A cube with volume 216 cm^3 is cut into three congruent pyramids. They share a tip at one corner of the cube, and each has an opposite face as its base. Find one pyramid's volume and explain your division.	**72 cm^3. The three congruent pyramids fill the cube, so each has 216/3 = 72 cm^3.**

PART 2 · APPLY IT

1. **78π cm^2** The unrolled side has area $2\pi(3)(10) = 60\pi$ cm^2. The two ends add $2\pi(3)^2 = 18\pi$ cm^2.
2. **64π cm^3** The matching cylinder has volume $\pi(4)^2(12) = 192\pi$ cm^3. The cone has one-third that volume, or 64π cm^3.

CHALLENGE

By Cavalieri's Principle, solids with the same height and equal areas at every horizontal slice have equal volume. The cylinder's volume is $\pi(6)^2(12) = 432\pi$ cm^3. Each cone's volume is $1/3\pi(6)^2(6) = 72\pi$ cm^3, so the sphere's volume is $432\pi - 2(72\pi) = 288\pi$ cm^3.

Accept equivalent exact expressions with π. For explanations, look for the stated volume comparison or the equal-slice reasoning, not just a formula.