LESSON MAKER PRO

Midpoint Proofs

Use coordinates to test and prove shape claims.

HIGH SCHOOL · GDA.GM.I

GDA.GM.I "Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases one may provide a more accessible or understandable argument than another."

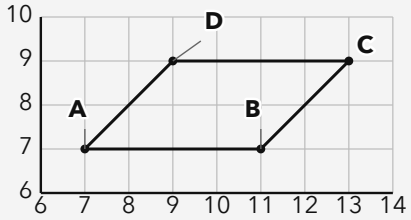
NAME _____ DATE _____

THE BIG IDEA

A quadrilateral is a parallelogram if its diagonals have the same midpoint. To find the midpoint of (a, b) and (c, d), average the x-coordinates and average the y-coordinates.

WORKED EXAMPLE

Is ABCD a parallelogram? Prove your answer using the figure.



1. Midpoint of AC: ((7 + 13)/2, (7 + 9)/2) = (10, 8).
2. Midpoint of BD: ((11 + 9)/2, (7 + 9)/2) = (10, 8).
3. The diagonals share a midpoint, so ABCD is a parallelogram.

Answer: Yes. Both diagonals have midpoint (10, 8).

PRACTICE 4 problems

1 A(0, 0), B(4, 0), C(6, 2), D(2, 2). Find the midpoint of AC.

ANSWER _____

2 A(-4, 0), B(0, 0), C(2, 4), D(-2, 4). Find the midpoint of BD.

ANSWER _____

3 A(0, 0), B(4, 0), C(5, 3), D(0, 3). Is ABCD a parallelogram? Use both diagonal midpoints.

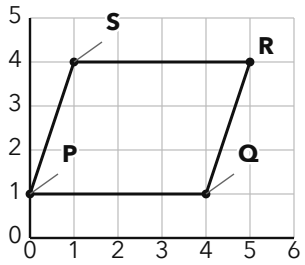
ANSWER _____

4 A(1, 1), B(5, 1), C(6, 4). Find D so ABCD is a parallelogram.

ANSWER _____

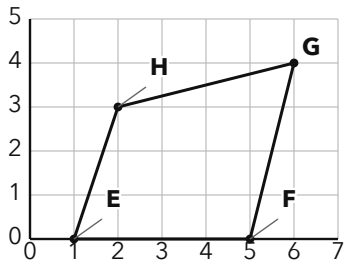
APPLY IT Show your work

1 A game designer marks a four-sided arena on a coordinate grid. Is PQRS a parallelogram? Prove it with midpoints.



ANSWER _____

2 A student sketches a four-sided logo for a club. Is EFGH a parallelogram? Prove your answer with midpoints.



ANSWER _____

CHALLENGE One step up

A, B, C, D are consecutive vertices of a parallelogram. A(-2, 1), B(2, 1), C(4, 5). Find D and the center of a half-turn that swaps A with C and B with D. Explain how the half-turn proves opposite sides parallel.

WORK SPACE AND ANSWER

ANSWER KEY

Midpoint Proofs

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A(0, 0), B(4, 0), C(6, 2), D(2, 2). Find the midpoint of AC.	**(3, 1)**
2	A(-4, 0), B(0, 0), C(2, 4), D(-2, 4). Find the midpoint of BD.	**(-1, 2)**
3	A(0, 0), B(4, 0), C(5, 3), D(0, 3). Is ABCD a parallelogram? Use both diagonal midpoints.	**No. AC has midpoint (5/2, 3/2), but BD has midpoint (2, 3/2).**
4	A(1, 1), B(5, 1), C(6, 4). Find D so ABCD is a parallelogram.	**(2, 4)**

PART 2 · APPLY IT

1. **Yes. PR and QS both have midpoint (5/2, 5/2).** Find the midpoint of each diagonal. The matching midpoints prove the arena is a parallelogram.
2. **No. EG has midpoint (7/2, 2), while FH has midpoint (7/2, 3/2).** The diagonal midpoints have different y-coordinates, so the logo is not a parallelogram.

CHALLENGE

D = (0, 5), and the center is (1, 3). The half-turn maps AB to CD and BC to DA. Each side is parallel to its image.

Accept either diagonal first in a midpoint proof. In the challenge, accept an equivalent explanation using a 180° rotation.