LESSON MAKER PRO

Find the Hidden Line

Turn exponential data into a line you can use.

HIGH SCHOOL · MM.D3D.B.16

MM.D3D.B.16 "Create a linear representation of non-linear data and interpret solutions, using technology and the process of linearization with logarithms."

NAME _____ DATE _____

LINEARIZE WITH LOGARITHMS

When y is a power of 2, $\log_2(y)$ is the exponent that gives y. A spreadsheet can graph x against $\log_2(y)$. On paper, find the exponents from powers of 2, then use the transformed points to write a line.

WORKED EXAMPLE

The table shows exponential data. Write a linear equation for $\log_2(y)$, then find y when x = 17.

EXPONENTIAL DATA

x	y
11	128
13	512
15	2048

1. The transformed points are (11, 7), (13, 9), and (15, 11).
2. Slope = (9 - 7)/(13 - 11) = 1.
3. Use (11, 7): 7 = 11 + b, so b = -4.
4. At x = 17, $\log_2(y) = 17 - 4 = 13$.

Answer: $\log_2(y) = x - 4$; $y = 2^{13} = 8192$

PRACTICE 6 problems

1 Find the slope of the line formed by the points $(x, \log_2(y))$.

DATA A

x	y
0	1
1	2
2	4

ANSWER _____

2 Write a linear equation for $\log_2(y)$ in terms of x.

DATA B

x	y
0	4
1	8
2	16

ANSWER _____

3 Use a linear equation for $\log_2(y)$ to find x when y = 64.

DATA C

x	y
0	1
1	4
2	16

ANSWER _____

4 Write a linear equation for $\log_2(y)$ in terms of x.

DATA D

x	y
0	16
1	8
2	4

ANSWER _____

5 A data set has the linear representation $\log_2(y) = 2x + 1$. Find y when x = 2.

ANSWER _____

6 Write a linear equation for $\log_2(y)$, then find x when y = 16.

DATA E

x	y
0	2
2	4
4	8

ANSWER _____

APPLY IT Show your work

1 A student's video has the view counts shown below. Let x be hours after posting and y be views. Write a linear equation for $\log_2(y)$, then predict the views after 4 hours.

VIDEO VIEWS

Hours (x)	Views (y)
0	8
1	16
2	32

ANSWER _____

2 A phone's battery charge follows the data below. Let x be hours and y be percent charge. Write a linear equation for $\log_2(y)$. According to this model, when will the charge reach 4%?

BATTERY CHARGE

Hours (x)	Charge, percent (y)
0	64
1	32
2	16

ANSWER _____

CHALLENGE One step up

Two fundraising pages start with different numbers of supporters. After x days, page A has 4×2^x supporters and page B has 2^{2x} supporters. Write a linear equation for $\log_2(y)$ for each page. Find when the pages have the same number of supporters, and explain why the lines meet then.

WORK SPACE AND ANSWER

ANSWER KEY

Find the Hidden Line

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the slope of the line formed by the points (x, $\log_2(y)$).	**1**
2	Write a linear equation for $\log_2(y)$ in terms of x.	**$\log_2(y) = x + 2$**
3	Use a linear equation for $\log_2(y)$ to find x when y = 64.	**x = 3**
4	Write a linear equation for $\log_2(y)$ in terms of x.	**$\log_2(y) = 4 - x$**
5	A data set has the linear representation $\log_2(y) = 2x + 1$. Find y when x = 2.	**y = 32**
6	Write a linear equation for $\log_2(y)$, then find x when y = 16.	**$\log_2(y) = x/2 + 1$; x = 6**

PART 2 · APPLY IT

1. **$\log_2(y) = x + 3$; 128 views** The transformed values are 3, 4, and 5, giving a line with slope 1 and intercept 3. At x = 4, $\log_2(y) = 7$, so y = 128.
2. **$\log_2(y) = 6 - x$; after 4 hours** The transformed values are 6, 5, and 4, so the line has slope -1 and intercept 6. Since $\log_2(4) = 2$, solve 2 = 6 - x to get x = 4.

CHALLENGE

Page A: $\log_2(y) = x + 2$. Page B: $\log_2(y) = 2x$. The lines meet at x = 2, when both pages have 16 supporters. Equal logarithms represent equal positive numbers of supporters.

Accept equivalent linear equations, such as $\log_2(y) = 1 + x/2$ in problem 6. For predictions, students should convert the logarithm back to the original y-value.