LESSON MAKER PRO

Count the Yeses

Build a probability distribution from a yes/no chance.

HIGH SCHOOL · MM.D3D.C.19.a

MM.D3D.C.19.a "Use the binomial theorem to construct the sampling distribution for the number of successes in a binary event or the number of positive responses to a yes/no question in a random sample."

NAME _____ DATE _____

THE BINOMIAL RULE

When each response is independent and has the same chance p of being yes, let X be the number of yes responses in n trials. For each possible count k, use $P(X = k) = C(n,k)p^k(1 - p)^{n-k}$; together, these probabilities form the distribution.

WORKED EXAMPLE

Two people answer independently. Each has a 1/4 chance of saying yes. Find the distribution of X, the number of yes answers.

WAYS TO GET EACH COUNT

k	0	1	2
C(2,k)	1	2	1

1. No answers: $P(X = 0) = 1(3/4)^2 = 9/16$.
2. One yes: $P(X = 1) = 2(1/4)(3/4) = 6/16$.
3. Two yes answers: $P(X = 2) = 1(1/4)^2 = 1/16$.

Answer: For k = 0, 1, 2, the probabilities are 9/16, 6/16, 1/16.

PRACTICE 4 problems

1 For 3 independent yes/no responses with P(yes) = 1/2, list P(X = k) for k = 0, 1, 2, 3.

ANSWER _____

2 For 4 independent yes/no responses with P(yes) = 1/2, list P(X = k) for k = 0 through 4.

ANSWER _____

3 For 3 independent yes/no responses with P(yes) = 1/3, list P(X = k) for k = 0 through 3.

ANSWER _____

4 For 4 independent yes/no responses with P(yes) = 1/3, list P(X = k) for k = 0 through 4.

ANSWER _____

APPLY IT Show your work

1 A school polls 4 randomly selected students about a later start time. Model their answers as independent, with each student having a 2/3 chance of saying yes. List the probabilities for 0 through 4 yes answers.

WORK SPACE

ANSWER _____

2 A school app surveys 3 randomly selected users about dark mode. Model their choices as independent, with each user having a 2/5 chance of choosing it. List the probabilities for 0 through 3 dark-mode choices.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Five independent voters each have a 1/2 chance of voting yes. List the distribution of X, the number of yes votes. Then find $P(X \geq 3)$.

WORK SPACE AND ANSWER

Count the Yeses

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For 3 independent yes/no responses with P(yes) = 1/2, list P(X = k) for k = 0, 1, 2, 3.	**1/8, 3/8, 3/8, 1/8**
2	For 4 independent yes/no responses with P(yes) = 1/2, list P(X = k) for k = 0 through 4.	**1/16, 4/16, 6/16, 4/16, 1/16**
3	For 3 independent yes/no responses with P(yes) = 1/3, list P(X = k) for k = 0 through 3.	**8/27, 12/27, 6/27, 1/27**
4	For 4 independent yes/no responses with P(yes) = 1/3, list P(X = k) for k = 0 through 4.	**16/81, 32/81, 24/81, 8/81, 1/81**

PART 2 · APPLY IT

1. **For 0 through 4 yes answers: 1/81, 8/81, 24/81, 32/81, 16/81.** Use $P(X = k) = C(4,k)(2/3)^k(1/3)^{4-k}$ for k = 0 through 4.
2. **For 0 through 3 dark-mode choices: 27/125, 54/125, 36/125, 8/125.** Use $P(X = k) = C(3,k)(2/5)^k(3/5)^{3-k}$ for k = 0 through 3.

CHALLENGE

For X = 0 through 5: 1/32, 5/32, 10/32, 10/32, 5/32, 1/32. P(X ≥ 3) = (10 + 5 + 1)/32 = 1/2.

Probabilities are listed in increasing order of the number of yes responses. Accept equivalent simplified fractions.