LESSON MAKER PRO

# Chances Add Up

*Build a distribution, then find the chance of a range of results.*

**HIGH SCHOOL · MM.D3D.C.19**

**MM.D3D.C.19** "Construct a sampling distribution for a random event or random sample. a. Use the binomial theorem to construct the sampling distribution for the number of successes in a binary event or the number of positive responses to a yes/no..."

NAME _____ DATE _____

## THE BINOMIAL RULE

For n independent yes/no trials with the same chance p of yes, the chance of exactly k yes results is $C(n,k)p^k(1 - p)^{n - k}$. Here, C(n,k) counts the ways to choose which trials give a yes result. Find the chance for each possible k to build a sampling distribution, then add chances to find the probability of a range.

## WORKED EXAMPLE

A game gives a prize on each play with probability 1/5. Plays are independent. What is the probability of exactly 4 prizes in 5 plays?

1. Choose which 4 of the 5 plays give prizes: C(5,4) = 5.
2. Find the probability of one such order: $(1/5)^4(4/5) = 4/3125$.
3. Multiply by the number of orders: 5 × 4/3125 = 4/625.

**Answer: 4/625**

## PRACTICE 6 problems

**1** A yes/no result has a 1/2 chance of yes on each of 2 independent trials. List the probabilities of 0, 1, and 2 yes results, in that order.

**ANSWER** _____

**2** A yes/no result has a 1/2 chance of yes on each of 3 independent trials. What is the probability of exactly 2 yes results?

**ANSWER** _____

**3** A yes/no result has a 1/2 chance of yes on each of 4 independent trials. What is the probability of at least 3 yes results?

**ANSWER** _____

**4** A yes/no result has a 1/4 chance of yes on each of 2 independent trials. What is the probability of exactly 1 yes result?

**ANSWER** _____

**5** A yes/no result has a 1/4 chance of yes on each of 3 independent trials. What is the probability of at most 1 yes result?

**ANSWER** _____

**6** A yes/no result has a 3/4 chance of yes on each of 4 independent trials. What is the probability of exactly 3 yes results?

**ANSWER** _____

## APPLY IT Show your work

**1** A school surveys 3 randomly chosen students about adding a music room. For this model, each student independently has a 2/3 chance of saying yes. What is the probability that at least 2 students say yes?

WORK SPACE

**ANSWER** _____

**2** An app offers a bonus on each of 4 days. Each day independently has a 1/2 chance of a bonus. What is the probability of getting at least one bonus?

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

A digital game gives a rare item on each of 4 independent tries with probability 1/4 per try. List the probabilities of getting 0, 1, 2, 3, and 4 items. Then decide which is more likely: getting no items or getting at least 2 items. Show the probabilities you compared.

WORK SPACE AND ANSWER

# Chances Add Up



## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | A yes/no result has a 1/2 chance of yes on each of 2 independent trials. List the probabilities of 0, 1, and 2 yes results, in that order. | **0: 1/4; 1: 1/2; 2: 1/4** |
| 2 | A yes/no result has a 1/2 chance of yes on each of 3 independent trials. What is the probability of exactly 2 yes results? | **3/8** |
| 3 | A yes/no result has a 1/2 chance of yes on each of 4 independent trials. What is the probability of at least 3 yes results? | **5/16** |
| 4 | A yes/no result has a 1/4 chance of yes on each of 2 independent trials. What is the probability of exactly 1 yes result? | **3/8** |
| 5 | A yes/no result has a 1/4 chance of yes on each of 3 independent trials. What is the probability of at most 1 yes result? | **27/32** |
| 6 | A yes/no result has a 3/4 chance of yes on each of 4 independent trials. What is the probability of exactly 3 yes results? | **27/64** |

## PART 2 · APPLY IT

1. **20/27** Add the probabilities of exactly 2 and exactly 3 yes responses: $C(3,2)(2/3)^2(1/3) + (2/3)^3 = 12/27 + 8/27 = 20/27$.
2. **15/16** The only outcome outside this range is no bonuses. Its probability is $(1/2)^4 = 1/16$, so the requested probability is $1 - 1/16 = 15/16$.

## CHALLENGE

**For 0 through 4 items, the probabilities are 81/256, 108/256, 54/256, 12/256, and 1/256. No items is more likely: 81/256 compared with (54 + 12 + 1)/256 = 67/256.**

Accept equivalent fractions. In the challenge, the five probabilities should sum to 1.