LESSON MAKER PRO

Chance or Evidence?

Use exact p-values to test claims about chance.

HIGH SCHOOL · MM.D3D.C.20.c

MM.D3D.C.20.c "Perform a significance test for null and alternative hypotheses."

NAME _____ DATE _____

TEST THE CLAIM

For independent trials with two equally likely outcomes, there are 2^n possible sequences. To find a one-sided p-value, count the sequences at least as extreme as your result in the direction of H_a, then divide by 2^n. Reject H_0 when the p-value is at most alpha; otherwise, do not reject H_0.

WORKED EXAMPLE

A coin is flipped 7 times, and all 7 flips are heads. Test H_0: $p = 1/2$ against H_a: $p > 1/2$ at alpha = 1/10, where p is the chance of heads. Find the exact p-value and make a decision.

1. There are $2^7 = 128$ equally likely sequences.
2. Only one sequence has 7 or more heads, so the p-value is 1/128.
3. Because $1/128 < 1/10$, reject H_0.

Answer: p-value = 1/128; reject H_0.

PRACTICE 6 problems

1 A coin is flipped 4 times, and all 4 flips are heads. Test H_0: $p = 1/2$ against H_a: $p > 1/2$ at alpha = 1/8. Find the exact p-value and decide whether to reject H_0.

ANSWER _____

2 A coin is flipped 4 times, and 3 flips are heads. Test H_0: $p = 1/2$ against H_a: $p > 1/2$ at alpha = 1/4. Find the exact p-value and decide whether to reject H_0.

ANSWER _____

3 A coin is flipped 6 times, and no flips are heads. Test H_0: $p = 1/2$ against H_a: $p < 1/2$ at alpha = 1/20. Find the exact p-value and decide whether to reject H_0.

ANSWER _____

4 A coin is flipped 6 times, and 1 flip is heads. Test H_0: $p = 1/2$ against H_a: $p < 1/2$ at alpha = 1/8. Find the exact p-value and decide whether to reject H_0.

ANSWER _____

5 A coin is flipped 8 times, and 7 flips are heads. Test H_0: $p = 1/2$ against H_a: $p > 1/2$ at alpha = 1/20. Find the exact p-value and decide whether to reject H_0.

ANSWER _____

6 A coin is flipped 8 times, and 2 flips are heads. Test H_0: $p = 1/2$ against H_a: $p < 1/2$ at alpha = 1/8. Find the exact p-value and decide whether to reject H_0.

ANSWER _____

APPLY IT Show your work

1 A student guesses independently on 10 quiz questions, each with two answer choices. The student gets 9 correct and claims to be doing better than chance. Let p be the chance of a correct answer. Test H_0: p = 1/2 against H_a: p > 1/2 at alpha = 1/20. Find the exact p-value and decide whether the results support the claim.

WORK SPACE

ANSWER _____

2 A friend claims to recognize which of two artists made a song after hearing a short clip. On 6 independent trials, the friend gets 4 correct. Assume guessing gives a 1/2 chance of being correct on each trial. Let p be the chance of a correct answer. Test H_0: p = 1/2 against H_a: p > 1/2 at alpha = 1/4. Find the exact p-value and decide whether the results support the claim.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A coin is flipped 6 times, and none of the flips are heads. Test H_0: p = 1/2 against H_a: p is not equal to 1/2 at alpha = 1/20. For this two-sided test, results as far from 3 heads as the observed result count toward the p-value. Find the exact p-value, make a decision, and explain which results you counted.

WORK SPACE AND ANSWER

Chance or Evidence?

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A coin is flipped 4 times, and all 4 flips are heads. Test H_0: $p = 1/2$ against H_a: $p > 1/2$ at alpha = 1/8. Find the exact p-value and decide whether to reject H_0.	**p-value = 1/16; reject H_0.**
2	A coin is flipped 4 times, and 3 flips are heads. Test H_0: $p = 1/2$ against H_a: $p > 1/2$ at alpha = 1/4. Find the exact p-value and decide whether to reject H_0.	**p-value = 5/16; do not reject H_0.**
3	A coin is flipped 6 times, and no flips are heads. Test H_0: $p = 1/2$ against H_a: $p < 1/2$ at alpha = 1/20. Find the exact p-value and decide whether to reject H_0.	**p-value = 1/64; reject H_0.**
4	A coin is flipped 6 times, and 1 flip is heads. Test H_0: $p = 1/2$ against H_a: $p < 1/2$ at alpha = 1/8. Find the exact p-value and decide whether to reject H_0.	**p-value = 7/64; reject H_0.**
5	A coin is flipped 8 times, and 7 flips are heads. Test H_0: $p = 1/2$ against H_a: $p > 1/2$ at alpha = 1/20. Find the exact p-value and decide whether to reject H_0.	**p-value = 9/256; reject H_0.**
6	A coin is flipped 8 times, and 2 flips are heads. Test H_0: $p = 1/2$ against H_a: $p < 1/2$ at alpha = 1/8. Find the exact p-value and decide whether to reject H_0.	**p-value = 37/256; do not reject H_0.**

PART 2 · APPLY IT

1. **p-value = 11/1024; reject H_0. The results support the claim at alpha = 1/20.** There are $2^{10} = 1024$ possible answer sequences. Ten sequences have exactly 9 correct, and one has 10 correct, so $11/1024 < 1/20$.
2. **p-value = 11/32; do not reject H_0. The results do not support the claim at alpha = 1/4.** Of the 64 possible sequences, 15 have exactly 4 correct, 6 have 5 correct, and 1 has 6 correct. Thus the p-value is $22/64 = 11/32$, which is greater than 1/4.

CHALLENGE

p-value = 2/64 = 1/32; reject H_0. Count 0 heads and 6 heads because both are 3 heads away from 3. Each occurs in one of the 64 possible sequences, and $1/32 < 1/20$.

Accept equivalent exact fractions and decisions that match the p-values. For the challenge, students must count both extreme outcomes, not just the observed outcome.