LESSON MAKER PRO

Rate Reality Check

Compare what an advertised rate earns in a year.

HIGH SCHOOL · MM.FPM.A.4.a

MM.FPM.A.4.a "Explain the relationship between annual percentage yield (APY) and annual percentage rate (APR) as values for r in the formulas $A=P(1+r)^t$ and $A=Pe^{rt}$."

NAME _____ DATE _____

TWO WAYS TO STATE A RATE

In $A=P(1+r)^t$, r is the APY, the rate the money actually earns in one year. In $A=Pe^{rt}$, r is the APR for continuous compounding. For the formulas to give the same balance, $1+APY=e^{APR}$, so $APY=e^{APR}-1$ and $APR=\ln(1+APY)$.

WORKED EXAMPLE

An account starts with \$28 and has a continuously compounded APR of $\ln(3/2)$. Find its APY and its balance after 2 years.

1. One-year growth factor: $e^{\ln(3/2)}=3/2$.
2. $APY=3/2-1=1/2$.
3. $A=28(1+1/2)^2=28(9/4)=63$.

Answer: APY=1/2; balance=\$63

PRACTICE 6 problems

1 A continuously compounded APR is $\ln(5/4)$. Find the APY.

ANSWER _____

2 An account has an APY of 1/5. Find its equivalent continuously compounded APR.

ANSWER _____

3 An account starts with \$27 and has a continuously compounded APR of $\ln(4/3)$. Find its APY and its balance after 3 years.

ANSWER _____

4 An account starts with \$81 and has an APY of 1/9. Find its equivalent continuously compounded APR and its balance after 1 year.

ANSWER _____

5 Account A offers an APY of 1/8. Account B offers a continuously compounded APR of $\ln(9/8)$. Which account earns more after 1 year? Give a short reason.

ANSWER _____

6 An account starts with \$100 and has a continuously compounded APR of $\ln(11/10)$. Find its APY and the interest earned after 1 year.

ANSWER _____

APPLY IT Show your work

1 You put $250 into a concert fund that earns an APY of 1/25. What continuously compounded APR would earn the same amount in 1 year? What would your balance be then?

WORK SPACE

ANSWER _____

2 You put $343 into a bike fund with a continuously compounded APR of ln(8/7). What is its APY, and what will the balance be after 3 years if the rate stays the same?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Two accounts start with the same amount. Account A has an APY of 1/3. Account B has a continuously compounded APR of ln(16/9)/2. Will their balances be equal after 2 years? Explain using the growth factors in both formulas.

WORK SPACE AND ANSWER

ANSWER KEY

Rate Reality Check

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A continuously compounded APR is ln(5/4). Find the APY.	**1/4**
2	An account has an APY of 1/5. Find its equivalent continuously compounded APR.	**ln(6/5)**
3	An account starts with $27 and has a continuously compounded APR of ln(4/3). Find its APY and its balance after 3 years.	**APY=1/3; balance=$64**
4	An account starts with $81 and has an APY of 1/9. Find its equivalent continuously compounded APR and its balance after 1 year.	**APR=ln(10/9); balance=$90**
5	Account A offers an APY of 1/8. Account B offers a continuously compounded APR of ln(9/8). Which account earns more after 1 year? Give a short reason.	**Neither. Both have a one-year growth factor of 9/8.**
6	An account starts with $100 and has a continuously compounded APR of ln(11/10). Find its APY and the interest earned after 1 year.	**APY=1/10; interest=$10**

PART 2 · APPLY IT

1. **APR=ln(26/25); balance=$260** The one-year growth factor is 1+1/25=26/25, so the equivalent APR is ln(26/25). The balance is $250(26/25)=$260.
2. **APY=1/7; balance=$512** The one-year growth factor is $e^{\ln(8/7)}=8/7$, so the APY is 1/7. After 3 years, the balance is $343(8/7)^3=$512.

CHALLENGE

Yes. Account A has a 2-year growth factor of $(4/3)^2=16/9$. Account B has a 2-year growth factor of $e^{(\ln(16/9)/2)(2)}=16/9$.

Accept equivalent exact expressions. Do not require decimal approximations of logarithms.