LESSON MAKER PRO

Sums That Stretch

Find finite sums and explore what happens when a pattern continues forever.

HIGH SCHOOL · PC.A.A.15

PC.A.A.15 "Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems, extending to infinite geometric series."

NAME _____ **DATE** _____

CANCEL TO FIND THE SUM

Call a geometric sum S_n. Multiply it by the common ratio r, then subtract: the middle terms cancel. If the first term is a and $r \neq 1$, $S_n = a(1 - r^n)/(1 - r)$. When $|r| < 1$, r^n approaches 0, so the infinite sum is $a/(1 - r)$.

WORKED EXAMPLE

Derive and find the sum of the first 3 terms when a = 11 and r = 1/5.

1. Write $S_3 = 11 + 11/5 + 11/25$.
2. Multiply: $(1/5)S_3 = 11/5 + 11/25 + 11/125$.
3. Subtract: $(4/5)S_3 = 11 - 11/125 = 1364/125$.
4. Divide: $S_3 = (1364/125)/(4/5) = 341/25$.

Answer: 341/25

PRACTICE 6 problems

1 Find the sum of the first 5 terms when a = 2 and r = 3.

ANSWER _____

2 Find the sum of the first 4 terms when a = 4 and r = -2.

ANSWER _____

3 Find the sum: 6 + 3 + 3/2 + 3/4.

ANSWER _____

4 Find the sum of the first 2 terms when a = 12 and r = 1/4.

ANSWER _____

5 Find the infinite sum when a = 9 and r = 2/3.

ANSWER _____

6 Find the infinite sum when a = 8 and r = -1/2.

ANSWER _____

APPLY IT Show your work

1 A video gets 640 views on day 1. On each of the next 4 days, it gets half as many views as the day before. How many views does it get in all 5 days?

WORK SPACE

ANSWER _____

2 A game update downloads 12 MB in its first second. In each later second, it downloads 3/4 as much data as in the previous second. If this pattern continues forever, how much data does the model predict will download?

WORK SPACE

ANSWER _____

CHALLENGE One step up

An infinite geometric series has a sum of 24. Its first 2 terms have a sum of 18, and $0 < r < 1$. Find the first term and the common ratio. Explain your reasoning.

WORK SPACE AND ANSWER

Sums That Stretch

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the sum of the first 5 terms when a = 2 and r = 3.	**242**
2	Find the sum of the first 4 terms when a = 4 and r = -2.	**-20**
3	Find the sum: 6 + 3 + 3/2 + 3/4.	**45/4**
4	Find the sum of the first 2 terms when a = 12 and r = 1/4.	**15**
5	Find the infinite sum when a = 9 and r = 2/3.	**27**
6	Find the infinite sum when a = 8 and r = -1/2.	**16/3**

PART 2 · APPLY IT

1. **1,240 views** The five daily counts are 640, 320, 160, 80, and 40. Their sum is 1,240.
2. **48 MB** The infinite series has a = 12 and r = 3/4. Its sum is 12/(1 - 3/4) = 48 MB.

CHALLENGE

The first term is 12, and r = 1/2. Since the first two terms make up 1 - r^2 of the infinite sum, 18/24 = 1 - r^2, so r^2 = 1/4. The condition 0 < r < 1 gives r = 1/2, and the first term is 24(1 - 1/2) = 12.

Accept equivalent exact fractions or mixed numbers. In the challenge, the explanation should use 0 < r < 1 to rule out r = -1/2.