LESSON MAKER PRO

Remainder Reveals

Use one substitution to find a remainder and test a factor.

HIGH SCHOOL · PC.A.B.16

PC.A.B.16 "Derive and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x)."

NAME _____ DATE _____

THE BIG IDEA

When p(x) is divided by x-a, you can write p(x)=(x-a)q(x)+r. Substitute a for x, and the first term becomes 0, so the remainder is p(a). This means x-a is a factor exactly when p(a)=0.

WORKED EXAMPLE

Find the remainder when $p(x)=x^2+13x+17$ is divided by x-8. Is x-8 a factor?

1. The divisor x-8 tells you to find p(8).
2. $p(8)=8^2+13(8)+17$
3. $p(8)=64+104+17=185$

Answer: The remainder is 185. Since p(8) is not 0, x-8 is not a factor.

PRACTICE 6 problems

1 Find the remainder when $p(x)=x^2-3x+2$ is divided by x-1.

ANSWER _____

2 Find the remainder when $p(x)=2x^3-x+3$ is divided by x+2.

ANSWER _____

3 Is x+1 a factor of $p(x)=3x^2+2x-1$? Give the value that proves your answer.

ANSWER _____

4 Find the remainder when $p(x)=x^3-2x^2+x-3$ is divided by x-2.

ANSWER _____

5 Find c so that x-2 is a factor of $p(x)=cx^2+3x-10$.

ANSWER _____

6 For $p(x)=x^3-4x$, which is a factor: x-2, x+1, or both? Show the two values you checked.

ANSWER _____

APPLY IT Show your work

1 A student designs a rectangular gaming accessory box with side lengths x, x+1, and x+2 centimeters. Its volume is V(x)=x(x+1)(x+2) cubic centimeters. Find the remainder when V(x) is divided by x-3. What does the remainder mean for the box?

WORK SPACE

ANSWER _____

2 A student selling custom phone charms models profit, in dollars, by $P(x)=x^2+3x-10$, where x is the number of charms sold. Is x-2 a factor of P(x)? Explain what your answer means when 2 charms are sold.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find c and d so that both x-1 and x+2 are factors of $p(x)=x^3+cx^2+dx+2$. Show the two equations you use.

WORK SPACE AND ANSWER

Remainder Reveals

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the remainder when $p(x)=x^2-3x+2$ is divided by $x-1$.	**0**
2	Find the remainder when $p(x)=2x^3-x+3$ is divided by $x+2$.	**-11**
3	Is $x+1$ a factor of $p(x)=3x^2+2x-1$? Give the value that proves your answer.	**Yes. p(-1)=3-2-1=0.**
4	Find the remainder when $p(x)=x^3-2x^2+x-3$ is divided by $x-2$.	**-1**
5	Find c so that $x-2$ is a factor of $p(x)=cx^2+3x-10$.	**c=1**
6	For $p(x)=x^3-4x$, which is a factor: $x-2$, $x+1$, or both? Show the two values you checked.	**Only x-2 is a factor. p(2)=0 and p(-1)=3.**

PART 2 · APPLY IT

1. **The remainder is 60. When x=3, the box has a volume of 60 cubic centimeters.** $V(3)=3(4)(5)=60$. By the Remainder Theorem, the remainder on division by $x-3$ is $V(3)$.
2. **Yes. Selling 2 charms gives a profit of $0, so the student breaks even.** $P(2)=2^2+3(2)-10=0$. Therefore $x-2$ is a factor, and the profit at 2 charms is $0.

CHALLENGE

p(1)=0 gives c+d=-3. p(-2)=0 gives 4c-2d=6. Solving gives c=0 and d=-3.

Accept an evaluation such as $p(a)=0$ as justification that a divisor is a factor. In the challenge, equivalent equations and any correct solution method are acceptable.