LESSON MAKER PRO

Rates in Motion

Find average rates of change with difference quotients.

HIGH SCHOOL · PC.F.A.25.a

PC.F.A.25.a "Find the difference quotient $(f(x+\Delta x)-f(x))/(\Delta x)$ of a function and use it to evaluate the average rate of change at a point."

NAME _____ **DATE** _____

THE BIG IDEA

The difference quotient, $(f(x+\Delta x)-f(x))/\Delta x$, gives the average rate of change from x to $x+\Delta x$, where Δx is not 0. Substitute $x+\Delta x$ into the function, subtract f(x), and simplify before evaluating at the given values.

WORKED EXAMPLE

For $f(x)=7x^2+11x$, find the difference quotient. Then evaluate it at $x=3$ and $\Delta x=4$.

1. $f(x+\Delta x)=7x^2+14x\Delta x+7(\Delta x)^2+11x+11\Delta x$
2. $f(x+\Delta x)-f(x)=14x\Delta x+7(\Delta x)^2+11\Delta x$
3. $(f(x+\Delta x)-f(x))/\Delta x=14x+7\Delta x+11$
4. At $x=3$ and $\Delta x=4$: $14(3)+7(4)+11=81$

Answer: Difference quotient: $14x+7\Delta x+11$; average rate of change: 81

PRACTICE 6 problems

1 Find the difference quotient for $f(x)=x^2+2x$.

ANSWER _____

2 For $f(x)=2x^2+x$, find the difference quotient. Then evaluate it at $x=1$ and $\Delta x=2$.

ANSWER _____

3 For $f(x)=6x^2-2x$, find the difference quotient. Then evaluate it at $x=0$ and $\Delta x=1$.

ANSWER _____

4 Find the difference quotient for $f(x)=(x-2)^2$.

ANSWER _____

5 Find the difference quotient for $f(x)=x^3$.

ANSWER _____

6 For $f(x)=8-2x^2$, find the difference quotient. Then evaluate it at $x=1$ and $\Delta x=-2$.

ANSWER _____

APPLY IT Show your work

1 A video's view count is modeled by $V(t)=6t^2+10t$, where t is hours after posting. Use the difference quotient to find the average change in views per hour from t=2 to t=2+1. State what the result means.

WORK SPACE

ANSWER _____

2 A phone's battery level is modeled by $B(t)=100-2t^2-6t$, where t is hours since unplugging and B(t) is a percent. Use the difference quotient to find the average change in battery level per hour from t=1 to t=1+2. Explain the sign of your answer.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x)=1/(x+2)$, find the difference quotient and evaluate it at $x=0$ and $\Delta x=2$. Explain in one sentence why the value is negative.

WORK SPACE AND ANSWER

Rates in Motion

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the difference quotient for $f(x)=x^2+2x$.	**$2x+\Delta x+2$**
2	For $f(x)=2x^2+x$, find the difference quotient. Then evaluate it at $x=1$ and $\Delta x=2$.	**Difference quotient: $4x+2\Delta x+1$; value: 9**
3	For $f(x)=6x^2-2x$, find the difference quotient. Then evaluate it at $x=0$ and $\Delta x=1$.	**Difference quotient: $12x+6\Delta x-2$; value: 4**
4	Find the difference quotient for $f(x)=(x-2)^2$.	**$2x+\Delta x-4$**
5	Find the difference quotient for $f(x)=x^3$.	**$3x^2+3x\Delta x+(\Delta x)^2$**
6	For $f(x)=8-2x^2$, find the difference quotient. Then evaluate it at $x=1$ and $\Delta x=-2$.	**Difference quotient: $-4x-2\Delta x$; value: 0**

PART 2 · APPLY IT

1. **40 views per hour; the video gains an average of 40 views per hour during that hour.** The difference quotient is $12t+6\Delta t+10$. At $t=2$ and $\Delta t=1$, it equals 40.
2. **−14 percentage points per hour; the negative sign means the battery level is falling.** The difference quotient is $-4t-2\Delta t-6$. At $t=1$ and $\Delta t=2$, it equals −14.

CHALLENGE

Difference quotient: $-1/((x+2)(x+\Delta x+2))$; value: $-1/8$. The value is negative because f falls from 1/2 to 1/4 as x rises from 0 to 2.

Accept equivalent simplified forms of each difference quotient. In the battery problem, accept an average decrease of 14 percentage points per hour if the direction is clear.