LESSON MAKER PRO

Closer and Closer

Use shrinking intervals to estimate a rate at one point.

HIGH SCHOOL · PC.F.A.25.b

PC.F.A.25.b "Explore how the average rate of change of a function over an interval (presented symbolically or as a table) can be used to approximate the instantaneous rate of change at a point as the interval decreases."

NAME _____ DATE _____

FROM AVERAGE TO INSTANTANEOUS

The average rate of change from $x = a$ to $x = b$ is $(f(b) - f(a))/(b - a)$. To estimate the instantaneous rate at a, find average rates over intervals that get shorter while keeping a as one endpoint. The rate from the shortest interval is an estimate, not necessarily the exact rate.

WORKED EXAMPLE

For $f(x) = x^2$, find the average rates on $[7, 36/5]$ and $[7, 141/20]$. Use the shorter interval to estimate the instantaneous rate at $x = 7$.

1. For $f(x) = x^2$, $(f(b) - f(7))/(b - 7) = (b^2 - 49)/(b - 7) = b + 7$.
2. On $[7, 36/5]$, the rate is $36/5 + 7 = 71/5$.
3. On $[7, 141/20]$, the rate is $141/20 + 7 = 281/20$.
4. The second interval is shorter, so use its rate as the estimate.

Answer: 71/5 and 281/20; estimated instantaneous rate: 281/20

PRACTICE 6 problems

1 For $f(x) = x^2$, find the average rates on $[2, 4]$ and $[2, 5/2]$. Use the shorter interval to estimate the instantaneous rate at $x = 2$.

ANSWER _____

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2 Use the table to find the average rates on $[0, 1]$ and $[0, 1/2]$. Use the shorter interval to estimate the instantaneous rate at $t = 0$.

VALUES OF G(T)

t	g(t)
0	0
1/2	5/4
1	3

ANSWER _____

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3 For $f(x) = x^2 + x$, find the average rates on [1, 3] and [5/2, 3]. Use the shorter interval to estimate the instantaneous rate at $x = 3$.

ANSWER _____

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4 For $f(x) = 5x - 2$, find the average rates on [1, 3] and [1, 3/2]. What do the rates tell you about the instantaneous rate at $x = 1$?

ANSWER _____

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5 Use the table to find the average rates on [5, 6] and [11/2, 6]. Use the shorter interval to estimate the instantaneous rate at $x = 6$.

VALUES OF H(X)

x	h(x)
5	2
11/2	5/4
6	1

ANSWER _____

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6 For $f(x) = 10 - x^2$, find the average rates on [0, 2] and [0, 1/4]. Use the shorter interval to estimate the instantaneous rate at $x = 0$.

ANSWER _____

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APPLY IT Show your work

1 A phone's battery percentage t hours after a full charge is modeled by $B(t) = 100 - 2t^2$. Find the average rates on [1, 2] and [1, 3/2]. Use the shorter interval to estimate how fast the battery percentage is changing at t = 1, including units.

WORK SPACE

ANSWER _____

2 The table shows the total views on a new video, t minutes after it was posted. Find the average rates on [10, 11] and [10, 21/2]. Use the shorter interval to estimate how fast the view count is changing at t = 10, including units.

VIDEO VIEWS

t (minutes)	Total views
10	260
21/2	279
11	300

ANSWER _____

CHALLENGE One step up

Let f(x) = |x - 1|. Find the average rates on [0, 1], [1/2, 1], [1, 3/2], and [1, 2]. As the intervals shrink toward x = 1, can they give one instantaneous rate at x = 1? Explain.

WORK SPACE AND ANSWER

Closer and Closer

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x) = x^2$, find the average rates on [2, 4] and [2, 5/2]. Use the shorter interval to estimate the instantaneous rate at x = 2.	**6 and 9/2; estimate: 9/2**
2	Use the table to find the average rates on [0, 1] and [0, 1/2]. Use the shorter interval to estimate the instantaneous rate at t = 0.	**3 and 5/2; estimate: 5/2**
3	For $f(x) = x^2 + x$, find the average rates on [1, 3] and [5/2, 3]. Use the shorter interval to estimate the instantaneous rate at x = 3.	**5 and 13/2; estimate: 13/2**
4	For f(x) = 5x - 2, find the average rates on [1, 3] and [1, 3/2]. What do the rates tell you about the instantaneous rate at x = 1?	**Both average rates are 5; the instantaneous rate is 5.**
5	Use the table to find the average rates on [5, 6] and [11/2, 6]. Use the shorter interval to estimate the instantaneous rate at x = 6.	**-1 and -1/2; estimate: -1/2**
6	For $f(x) = 10 - x^2$, find the average rates on [0, 2] and [0, 1/4]. Use the shorter interval to estimate the instantaneous rate at x = 0.	**-2 and -1/4; estimate: -1/4**

PART 2 · APPLY IT

1. **-6 and -5 percentage points per hour; estimated rate at t = 1: -5 percentage points per hour** B(1) = 98, B(2) = 92, and B(3/2) = 191/2. Divide each change in battery percentage by its change in hours.
2. **40 and 38 views per minute; estimated rate at t = 10: 38 views per minute** The view count increases by 40 over 1 minute and by 19 over 1/2 minute. Divide each increase by its time interval.

CHALLENGE

The rates are -1, -1, 1, and 1, in order. No single instantaneous rate exists at x = 1 because intervals from the left give -1 while intervals from the right give 1.

Accept equivalent fractions or decimals. For estimates, students should use the rate from the shorter interval and keep negative signs and context units where needed.