LESSON MAKER PRO

Conic Clues

Identify the curve from its equation.

HIGH SCHOOL · PC.F.D.31.b

PC.F.D.31.b "Identify the conic section that will be formed, given its equation in general form."

NAME _____ DATE _____

CHECK THE COEFFICIENTS

Write the equation as $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. For a real, nondegenerate conic, $B^2 - 4AC$ is negative for an ellipse, zero for a parabola, and positive for a hyperbola. If $B = 0$ and $A = C$, check whether the ellipse is a circle; the test does not classify equations that form lines or a point.

WORKED EXAMPLE

Identify the conic: $13x^2 + 17y^2 - 26x + 34y - 51 = 0$.

1. $A = 13$, $B = 0$, and $C = 17$.
2. $B^2 - 4AC = 0 - 4(13)(17) = -884$.
3. Completing the squares gives $13(x - 1)^2 + 17(y + 1)^2 = 81$, a real, nondegenerate conic.

Answer: Ellipse

PRACTICE 6 problems

1 Identify the conic: $x^2 + y^2 - 6x + 4y + 4 = 0$.

ANSWER _____

2 Identify the conic: $4x^2 + 9y^2 - 8x + 18y - 23 = 0$.

ANSWER _____

3 Identify the conic: $y^2 - 4x + 6y + 1 = 0$.

ANSWER _____

4 Identify the conic: $x^2 - y^2 - 4x - 2y - 6 = 0$.

ANSWER _____

5 Identify the conic: $x^2 + 4xy + y^2 - 6 = 0$.

ANSWER _____

6 Identify the conic: $x^2 + 2xy + y^2 - 8x + 8y = 0$.

ANSWER _____

APPLY IT Show your work

1 The edge of a spotlight on a concert floor follows $x^2 + y^2 - 10x + 6y + 18 = 0$, where x and y are measured in meters. What conic is the edge?

WORK SPACE

ANSWER _____

2 A camera app adds a curved overlay described by $x^2 - 6x - 4y + 1 = 0$ on its coordinate grid. What conic is the overlay?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Both equations have $B^2 - 4AC > 0$: (I) $x^2 - y^2 = 0$ and (II) $x^2 - y^2 - 9 = 0$. Which equation forms a hyperbola? Explain why the other does not.

WORK SPACE AND ANSWER

Conic Clues

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Identify the conic: $x^2 + y^2 - 6x + 4y + 4 = 0$.	**Circle**
2	Identify the conic: $4x^2 + 9y^2 - 8x + 18y - 23 = 0$.	**Ellipse**
3	Identify the conic: $y^2 - 4x + 6y + 1 = 0$.	**Parabola**
4	Identify the conic: $x^2 - y^2 - 4x - 2y - 6 = 0$.	**Hyperbola**
5	Identify the conic: $x^2 + 4xy + y^2 - 6 = 0$.	**Hyperbola**
6	Identify the conic: $x^2 + 2xy + y^2 - 8x + 8y = 0$.	**Parabola**

PART 2 · APPLY IT

1. **Circle** $A = C = 1$ and $B = 0$. Completing the squares gives $(x - 5)^2 + (y + 3)^2 = 16$, so the edge is a circle.
2. **Parabola** $A = 1$, $B = 0$, and $C = 0$, so $B^2 - 4AC = 0$. Completing the square gives $(x - 3)^2 = 4(y + 2)$, a parabola.

CHALLENGE

Equation (II) forms a hyperbola because $x^2 - y^2 = 9$. Equation (I) factors as $(x - y)(x + y) = 0$, so it forms two intersecting lines.

Accept circle as a special type of ellipse for the circle items. In the challenge, the explanation must identify equation (I) as two intersecting lines.