LESSON MAKER PRO

Loops on Grids

Find the path behind an equation.

HIGH SCHOOL · PC.F.E.32.a

PC.F.E.32.a "Graph parametric and polar equations."

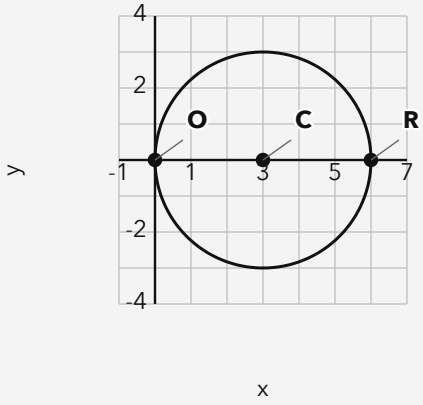
NAME _____ DATE _____

LOCATE THE CIRCLE

For $x=h+a\cos t$ and $y=k+a\sin t$, the graph is a circle with center (h,k) and radius $|a|$. For $r=a\cos\theta$ or $r=a\sin\theta$, use $x=r\cos\theta$, $y=r\sin\theta$, and $r^2=x^2+y^2$ to find the circle's center and radius.

WORKED EXAMPLE

How does the circle on the grid show the graph of $r=6\cos\theta$?



1. Multiply by r: $r^2=6r\cos\theta$.
2. Substitute: $x^2+y^2=6x$.
3. Complete the square: $(x-3)^2+y^2=9$.

Answer: The graph is the circle shown, with center C at (3,0) and radius 3.

PRACTICE 4 problems

1 Find the center and radius of $r=8\cos\theta$.

ANSWER _____

2 Find the center and radius of $r=10\sin\theta$.

ANSWER _____

3 For $x=1+3\cos t$ and $y=-2+3\sin t$, find the positions at $t=0$ and $t=\pi/2$.

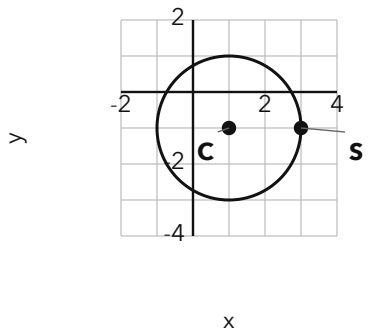
ANSWER _____

4 For $x=-3+5\cos t$ and $y=1+5\sin t$, find the center and radius.

ANSWER _____

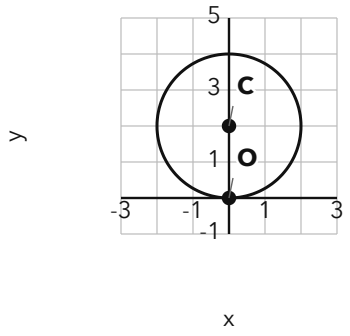
APPLY IT Show your work

1 A game character moves counterclockwise on the loop shown, starting at S when t=0. Write x(t) and y(t) for one lap, 0≤t≤2π.



ANSWER _____

2 A light show traces the loop shown, starting at O. Find its equation in the form r=a sin θ for 0≤θ≤π.



ANSWER _____

CHALLENGE One step up

For r=−10 sin θ, 0≤θ≤π, find the center and radius. Does this interval trace the whole circle? Explain briefly.

WORK SPACE AND ANSWER

Loops on Grids

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the center and radius of $r=8\cos\theta$.	**Center (4,0); radius 4.**
2	Find the center and radius of $r=10\sin\theta$.	**Center (0,5); radius 5.**
3	For $x=1+3\cos t$ and $y=-2+3\sin t$, find the positions at $t=0$ and $t=\pi/2$.	**At t=0: (4,−2). At t=π/2: (1,1).**
4	For $x=-3+5\cos t$ and $y=1+5\sin t$, find the center and radius.	**Center (−3,1); radius 5.**

PART 2 · APPLY IT

1. **$x(t)=1+2\cos t$; $y(t)=-1+2\sin t$, $0\le t\le 2\pi$.** The grid shows center (1,−1) and radius 2. Cosine starts at the rightmost point, and positive sine gives counterclockwise motion.
2. **$r=4\sin\theta$, $0\le\theta\le\pi$.** The circle has center (0,2) and radius 2, so $x^2+(y-2)^2=4$. Simplifying gives $r^2=4r\sin\theta$.

CHALLENGE

The center is (0,−5) and the radius is 5. Yes. The path goes from the origin through (0,−10) at $\theta=\pi/2$, then returns to the origin, tracing the whole circle.

Accept equivalent parametric notation and correct graphs if students sketch them. For the challenge, $r^2=-10r\sin\theta$ gives $x^2+(y+5)^2=25$.