LESSON MAKER PRO

Paths in Motion

Connect moving points, polar rules, and coordinate graphs.

HIGH SCHOOL · PC.F.E.32

PC.F.E.32 "Solve application-based problems involving parametric and polar equations. a. Graph parametric and polar equations. b. Convert parametric and polar equations to rectangular form."

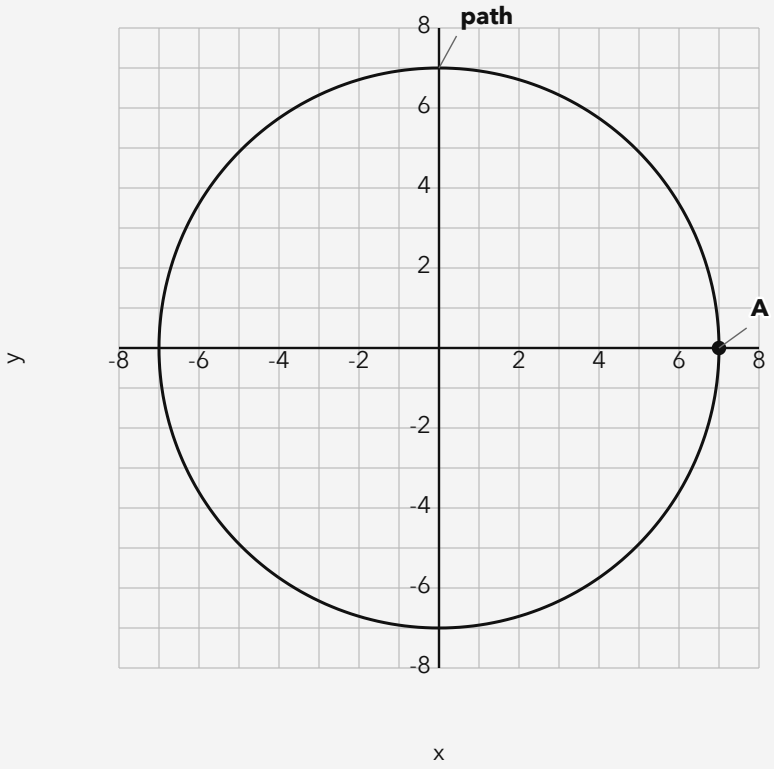
NAME _____ DATE _____

ONE PATH, TWO FORMS

To remove a parameter, solve one coordinate rule for the parameter and substitute into the other. To convert a polar rule, use $x = r\cos\theta$, $y = r\sin\theta$, and $r^2 = x^2 + y^2$. Check the graph or parameter interval to see which part of the curve is traced.

WORKED EXAMPLE

Use the graphed path to write a polar equation and a rectangular equation.



1. Point A is 7 units from the origin, and the path stays that distance from the origin.
2. Write the polar equation: $r = 7$.
3. Use $r^2 = x^2 + y^2$ to write the rectangular equation: $x^2 + y^2 = 49$.

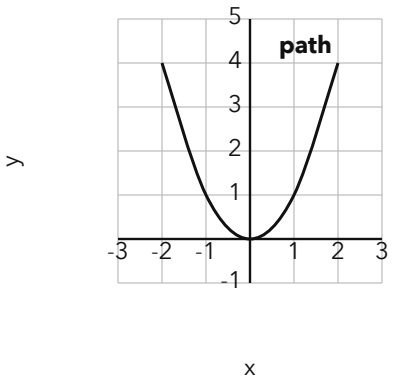
Answer: $r = 7$; $x^2 + y^2 = 49$

PRACTICE 6 problems

1 A point moves according to $x = 2t - 1$ and $y = t + 2$ for $-1 \le t \le 1$. Write the rectangular equation and the two endpoints of its path.

ANSWER _____

2 The graph shows the path of $x = t$ and $y = t^2$ for $-2 \le t \le 2$. Write its rectangular equation and identify the vertex on the graph.

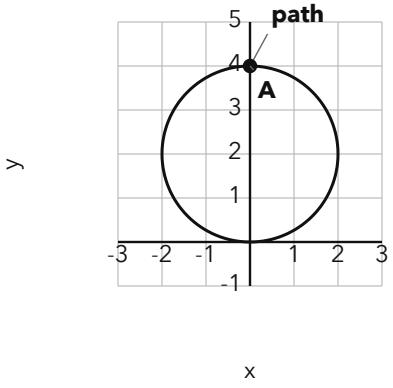


ANSWER _____

3 Convert $r = 2 \cos \theta$ to a rectangular equation.

ANSWER _____

4 Use the graphed circle to write its polar equation and its rectangular equation.



ANSWER _____

5 A point moves according to $x = t^2$ and $y = t$ for $-2 \le t \le 2$. Write the rectangular equation and state which side of the y-axis its path is on.

ANSWER _____

6 Convert $r \sin \theta = 3$ to a rectangular equation and name the type of graph.

ANSWER _____

APPLY IT Show your work

1 A drone filming a school event has coordinates $x = 2t + 1$ and $y = t + 1$, in meters, for $0 \le t \le 2$ seconds. Write a rectangular equation for its path and give its position after 2 seconds.

WORK SPACE

ANSWER _____

2 A location game marks the edge of a circular bonus zone with $r = 6 \cos \theta$. The player is at the origin, and the positive x-axis points east. Write the edge in rectangular form, then find its center and radius in meters.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A moving point follows $x = 2t$ and $y = t^2 - 1$ for $-2 \le t \le 2$. Write a rectangular equation for the path. The point (6, 8) satisfies that equation. Explain why the moving point still cannot reach it.

WORK SPACE AND ANSWER

Paths in Motion

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A point moves according to $x = 2t - 1$ and $y = t + 2$ for $-1 \le t \le 1$. Write the rectangular equation and the two endpoints of its path.	**$y = (x + 5)/2$; endpoints $(-3, 1)$ and $(1, 3)$**
2	The graph shows the path of $x = t$ and $y = t^2$ for $-2 \le t \le 2$. Write its rectangular equation and identify the vertex on the graph.	**$y = x^2$; vertex $(0, 0)$**
3	Convert $r = 2 \cos \theta$ to a rectangular equation.	**$x^2 + y^2 = 2x$**
4	Use the graphed circle to write its polar equation and its rectangular equation.	**$r = 4 \sin \theta$; $x^2 + (y - 2)^2 = 4$**
5	A point moves according to $x = t^2$ and $y = t$ for $-2 \le t \le 2$. Write the rectangular equation and state which side of the y-axis its path is on.	**$x = y^2$; on or to the right of the y-axis**
6	Convert $r \sin \theta = 3$ to a rectangular equation and name the type of graph.	**$y = 3$; horizontal line**

PART 2 · APPLY IT

1. **$y = (x + 1)/2$; (5, 3) meters** Solve $t = (x - 1)/2$ and substitute into $y = t + 1$. At $t = 2$, the coordinates are (5, 3).
2. **$(x - 3)^2 + y^2 = 9$; center (3, 0) meters; radius 3 meters** Multiply by r to get $r^2 = 6r \cos \theta$, so $x^2 + y^2 = 6x$. Completing the square gives the center and radius.

CHALLENGE

$y = x^2/4 - 1$. The allowed values of t give $-4 \le x \le 4$, so the point cannot reach $x = 6$.

Accept equivalent rectangular equations, including $x^2 + y^2 = 4y$ for problem 4. For the challenge, the explanation must use the parameter interval, not just the rectangular equation.