LESSON MAKER PRO

One Output, One Angle

Use a restricted interval to find a unique trigonometric angle.

HIGH SCHOOL · PC.F.G.35

PC.F.G.35 "Demonstrate that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed."

NAME _____ DATE _____

RESTRICT TO INVERT

A trigonometric function has an inverse on an interval where it is always increasing or always decreasing. On that interval, each output comes from exactly one input. Use radians, and check that your angle is in the given interval.

WORKED EXAMPLE

Find x if cos x = -sqrt(2)/2 and 0 <= x <= pi.

1. Cosine decreases throughout [0, pi], so the output has only one angle in this interval.
2. cos(3pi/4) = -sqrt(2)/2.
3. 3pi/4 is in [0, pi].

Answer: x = 3pi/4

PRACTICE 6 problems

1 Find x if sin x = 1/2 and -pi/2 <= x <= pi/2.

ANSWER _____

2 Find x if cos x = 1/2 and 0 <= x <= pi.

ANSWER _____

3 Find x if tan x = -1 and -pi/2 < x < pi/2.

ANSWER _____

4 Find x if sin x = 1/2 and pi/2 <= x <= 3pi/2.

ANSWER _____

5 Find x if cos x = 1/2 and pi <= x <= 2pi.

ANSWER _____

6 Give two different x-values in [0, 2pi] with sin x = 0. State why sine has no inverse on this whole interval.

ANSWER _____

APPLY IT Show your work

1 A 12-foot concert spotlight beam pivots upward. The height of its tip is h = 12 sin x feet, where 0 <= x <= pi/2. If the tip is 6 feet high, find x in radians and explain why the angle is unique.

WORK SPACE

ANSWER _____

2 A rotating camera rig has horizontal position d = 10 + 4 cos x feet, where pi <= x <= 2pi. If d = 8 feet, find x in radians and explain why the angle is unique.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let f(x) = sin x for pi/2 <= x <= 3pi/2. Write $f^{-1}(y)$ using arcsin(y), whose outputs lie in [-pi/2, pi/2]. Explain why your rule gives exactly one x for every -1 <= y <= 1. Then find $f^{-1}(-1/2)$.

WORK SPACE AND ANSWER

One Output, One Angle

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find x if sin x = 1/2 and -pi/2 <= x <= pi/2.	**x = pi/6**
2	Find x if cos x = 1/2 and 0 <= x <= pi.	**x = pi/3**
3	Find x if tan x = -1 and -pi/2 < x < pi/2.	**x = -pi/4**
4	Find x if sin x = 1/2 and pi/2 <= x <= 3pi/2.	**x = 5pi/6**
5	Find x if cos x = 1/2 and pi <= x <= 2pi.	**x = 5pi/3**
6	Give two different x-values in [0, 2pi] with sin x = 0. State why sine has no inverse on this whole interval.	**x = 0 and x = pi. Both give the same output, so sine is not one-to-one on [0, 2pi].**

PART 2 · APPLY IT

1. **x = pi/6. Sine increases throughout [0, pi/2], so this height occurs at only one angle.** Set 12 sin x = 6, giving sin x = 1/2. The angle in [0, pi/2] is pi/6.
2. **x = 4pi/3. Cosine increases throughout [pi, 2pi], so this position occurs at only one angle.** Set 10 + 4 cos x = 8, giving cos x = -1/2. The angle in [pi, 2pi] is 4pi/3.

CHALLENGE

$f^{-1}(y)$ = pi - arcsin(y), and $f^{-1}(-1/2)$ = 7pi/6. If a = arcsin(y), then sin(pi - a) = y and pi - a lies in [pi/2, 3pi/2]. Sine decreases throughout that interval, so no other x gives y.

Accept equivalent exact radian forms. In problem 6, any two distinct angles in the stated interval with sine 0 are valid if the explanation identifies the repeated output.