LESSON MAKER PRO

Identity Switch

Turn one Pythagorean identity into two useful forms.

HIGH SCHOOL · PC.F.H.37.a

PC.F.H.37.a "Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to derive the other forms of the identity."

NAME _____ DATE _____

DIVIDE EVERY TERM

Start with $\sin^2(\theta) + \cos^2(\theta) = 1$. Dividing every term by $\sin^2(\theta)$ or $\cos^2(\theta)$ creates an identity involving other trig ratios, but you can divide only when the divisor is not zero.

WORKED EXAMPLE

If $\sin(\theta) = 8/17$, use an identity to find $\csc^2(\theta)$.

1. $\sin^2(\theta) = 64/289$.
2. Divide $\sin^2(\theta) + \cos^2(\theta) = 1$ by $\sin^2(\theta)$: $1 + \cot^2(\theta) = \csc^2(\theta)$.
3. Since $\csc^2(\theta) = 1/\sin^2(\theta)$, $\csc^2(\theta) = 289/64$.

Answer: $\csc^2(\theta) = 289/64$

PRACTICE 6 problems

1 Divide $\sin^2(\theta) + \cos^2(\theta) = 1$ by $\cos^2(\theta)$. Write the resulting identity using $\tan(\theta)$ and $\sec(\theta)$, and state when the division is allowed.

ANSWER _____

2 If $\tan(\theta) = 2/3$, use a derived identity to find $\sec^2(\theta)$.

ANSWER _____

3 If $\cot(\theta) = 3/5$, use a derived identity to find $\csc^2(\theta)$.

ANSWER _____

4 If $\sec^2(\theta) = 10$, find $\tan^2(\theta)$.

ANSWER _____

5 If $\csc^2(\theta) = 17/9$, find $\cot^2(\theta)$.

ANSWER _____

6 If $\cos(\theta) = -4/7$, use a derived identity to find $\tan^2(\theta)$.

ANSWER _____

APPLY IT Show your work

1 A skateboard ramp rises 3 feet over a horizontal run of 4 feet. Let θ be the ramp's angle with the ground. For this ramp, $\sec^2(\theta)$ is the square of the ratio of ramp length to horizontal run. Use a derived identity to find $\sec^2(\theta)$ without finding the ramp length.

WORK SPACE

ANSWER _____

2 A drone is 12 meters above level ground and 5 meters horizontally from its operator. Let θ be the angle of elevation from the operator to the drone. Here, $\csc^2(\theta)$ is the square of the ratio of straight-line distance from the operator to the drone to the drone's height. Use a derived identity to find $\csc^2(\theta)$ without finding the straight-line distance.

WORK SPACE

ANSWER _____

CHALLENGE One step up

If $\csc^2(\theta) = 9/4$, find $\sec^2(\theta)$ without finding θ. Show how the Pythagorean identity connects the two values.

WORK SPACE AND ANSWER

ANSWER KEY # Identity Switch

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Divide $\sin^2(\theta) + \cos^2(\theta) = 1$ by $\cos^2(\theta)$. Write the resulting identity using tan(θ) and sec(θ), and state when the division is allowed.	**$1 + \tan^2(\theta) = \sec^2(\theta)$; $\cos(\theta) \neq 0$.**
2	If $\tan(\theta) = 2/3$, use a derived identity to find $\sec^2(\theta)$.	**13/9**
3	If $\cot(\theta) = 3/5$, use a derived identity to find $\csc^2(\theta)$.	**34/25**
4	If $\sec^2(\theta) = 10$, find $\tan^2(\theta)$.	**9**
5	If $\csc^2(\theta) = 17/9$, find $\cot^2(\theta)$.	**8/9**
6	If $\cos(\theta) = -4/7$, use a derived identity to find $\tan^2(\theta)$.	**33/16**

PART 2 · APPLY IT

1. **25/16** $\tan(\theta) = 3/4$, so $\sec^2(\theta) = 1 + \tan^2(\theta) = 1 + 9/16 = 25/16$.
2. **169/144** $\cot(\theta) = 5/12$, so $\csc^2(\theta) = 1 + \cot^2(\theta) = 1 + 25/144 = 169/144$.

CHALLENGE

$\sin^2(\theta) = 4/9$, so $\cos^2(\theta) = 1 - 4/9 = 5/9$. Therefore, $\sec^2(\theta) = 9/5$.

Accept equivalent forms of the derived identities. In the first practice problem, students must note that cos(θ) cannot equal zero.